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RWTH AACHEN UNIVERSITY

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Authors' addresses:

Andreas Knaut
RWTH Aachen University
Templergraben 55
52056 Aachen, Germany
E-mail: andreas.knaut@rwth-aachen.de

Reinhard Madlener, Christiane Rosen
Institute for Future Energy Consumer Needs and Behavior (FCN)
School of Business and Economics / E.ON Energy Research Center
RWTH Aachen University
Mathieustrasse 10
52074 Aachen, Germany
E-mail: RMadlener@eonerc.rwth-aachen.de, crosen@eonerc.rwth-aachen.de

Christian Vogt
Institute for Applied Geophysics and Geothermal Energy (GGE)
E.ON Energy Research Center
RWTH Aachen University
Mathieustrasse 10
52074 Aachen, Germany
E-mail: cvogt@eonerc.rwth-aachen.de

Publisher: Prof. Dr. Reinhard Madlener
Chair of Energy Economics and Management
Director, Institute for Future Energy Consumer Needs and Behavior (FCN)
E.ON Energy Research Center (E.ON ERC)
RWTH Aachen University
Mathieustrasse 10, 52074 Aachen, Germany
Phone: +49 (0) 241-80 49820
Fax: +49 (0) 241-80 49829
Web: www.eonerc.rwth-aachen.de/fcn
E-mail: post_fcn@eonerc.rwth-aachen.de

Effects of Temperature Uncertainty on the Valuation of Geothermal Projects: A Real Options Approach

Andreas Knaut*,[1], Reinhard Madlener[2], Christiane Rosen[2], and Christian Vogt[3]

[1]Institute of Energy Economics at the University of Cologne, Vogelsanger Str. 321, 50827 Cologne, Germany

[2]Future Energy Consumer Needs and Behavior (FCN), E.ON Energy Research Center, RWTH Aachen University, Mathieustr. 10, 52074 Aachen, Germany

[3]Applied Geophysics and Geothermal Energy (GGE), E.ON Energy Research Center, RWTH Aachen University, Mathieustr. 10, 52074 Aachen, Germany

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Abstract

In this paper we investigate the economic viability of geothermal district heating networks using both net present value (NPV) and real options analysis (ROA). We give an introduction to geothermal energy technologies and review the relevant ROA literature focusing on applications in similar fields, such as oil projects. The similarities and differences of oil and geothermal projects are discussed and summarized. The investment structure of geothermal projects is analyzed concerning costs and uncertainties in the different investment stages. We develop a method to assess the impact of temperature uncertainty with ROA. Investments in geothermal projects are evaluated applying a binomial and a trinomial lattice approach. A novel real options model for the evaluation of normally distributed uncertainty is developed for binomial lattices. Using data obtained from a Dutch project in The Hague, we find positive option values for both lattice approaches, compared to negative values for the common NPV calculation. Drilling and production costs are found to have a significant impact on the option value.

Keywords: Geothermal energy, Real options, District heating, Investment under uncertainty

1 Introduction

The foreseeable depletion of non-renewable energy resources and climate change induced through high anthropogenic emissions of carbon dioxide point out the importance of renewable energy sources for a sustainable energy supply in the future. Especially in

*Corresponding author. Tel. +49-221-277 29 306. E-mail: andreas.knaut@ewi.uni-koeln.de

Germany, the extension of renewable energy use plays an even more important role due to the planned nuclear power phase-out in 2022, and the unclear role of “clean coal” by means of carbon capture and storage (CCS) as a short- to medium-term remedy (Rohlfs and Madlener, 2013). The ambitious goal of the German government is to increase the share of renewables in electric power supply from 20% today to 35% in 2020 (Bundesregierung, 2011). This highlights the significance of cost-effective investments into renewable energy sources.

Whereas bioenergy, hydropower and wind energy belong to the established renewables in the market, geothermal energy so far only plays a minor role. Nevertheless, geothermal energy has the technical potential to satisfy the global heat and electricity demand, but the number of projects is still comparably low due to high costs which lead to a limited economic potential (Tester et al., 2006; IPCC, 2011). Compared to wind or solar energy, geothermal power is able to supply baseload power independently of weather conditions. Thus, it could take a significant part in a sustainable energy mix in the future.

Risks and uncertainties about geological conditions at the location are the main barriers for investors when assessing geothermal projects. Commonly used discounted cash flow or levelized cost-based investment evaluation models estimate risks but do not account for flexibility and irreversibility associated with the investment. Geothermal projects can be assessed and developed step-by-step, allowing investors to react to the arrival of new information. This flexibility should also be considered when evaluating geothermal investments.

Real options analysis (ROA) is a useful approach for the evaluation of investments, which is able to account for uncertainty, irreversibility and flexibility associated with a project. Applying this technique and considering multiple options that can be chosen during project realization, ROA is capable of calculating the option value of an investment that involves flexibility and uncertainty revelation. Since geothermal projects are subject to uncertainty, irreversibility and flexibility in the various investment stages, we use ROA in the present study in order to show its benefits and differences in results relative to conventional discounted cash flow analysis.

The remainder of this paper is organized as follows. Section 2 provides a brief introduction to geothermal energy resources and technologies. Section 3 describes the investment evaluation models considered, with a particular focus on ROA and the application to oil investments which have a similar project structure. The general procedure of geothermal investments and the associated costs and risks are explained in section 4. A valuation method for temperature uncertainty is developed and applied in a novel RO model for the modeling of normally distributed uncertainty (section 5). Section 6 presents the results obtained, while section 7 summarizes and concludes.

2 Geothermal energy use: significance and characteristics

Geothermal energy is a promising renewable energy source that can help to satisfy future energy demand in a sustainable manner. The overall theoretical and technical potentials are enormous, although of course only a fraction of geothermal resources can be harnessed cost-effectively today (IPCC, 2011; Clauser, 2006; Dickson and Fanelli, 2003). The deployment of geothermal energy can be divided into electricity generation and direct usage, which means heating in particular. In 2010, the installed capacity for electricity generation from geothermal systems worldwide was 11 GW_e, generating 67.2 TWh of electricity. The installed capacity for direct energy use was 51 GW_{th}, with an amount of 439 PJ of heat energy being supplied (REN21, 2011). The International Energy Agency projects an exponential increase of geothermal energy use, leading to a production of about 1400 TWh_e in 2050 (IEA, 2011).

The source of geothermal heat is located underneath the earth's surface. Depending on geological conditions and anomalies, temperature gradients from the surface to the earth's center vary largely at different locations. The average gradient near the surface is about 30 $K\,km^{-1}$ (Barbier, 2002). At the beginning of geothermal energy exploitation, the aim is to identify locations with high temperature gradients, where heat can be extracted from the ground with relatively low effort at shallow depths. Some temperature gradients, which result from the exploration at different sites in France, Germany and Italy, are shown in fig. 1. The gradients vary with depth due to changes in rock properties such as thermal conductivity. Ground properties, therefore, have to be evaluated carefully when assessing geothermal projects. This shows that at the beginning of a geothermal project, there are typically large uncertainties associated with temperature estimation.

Possible systems for geothermal energy use include hydrothermal systems, enhanced geothermal systems (EGS), geopressured systems, and magmatic systems (DiPippo, 2008).

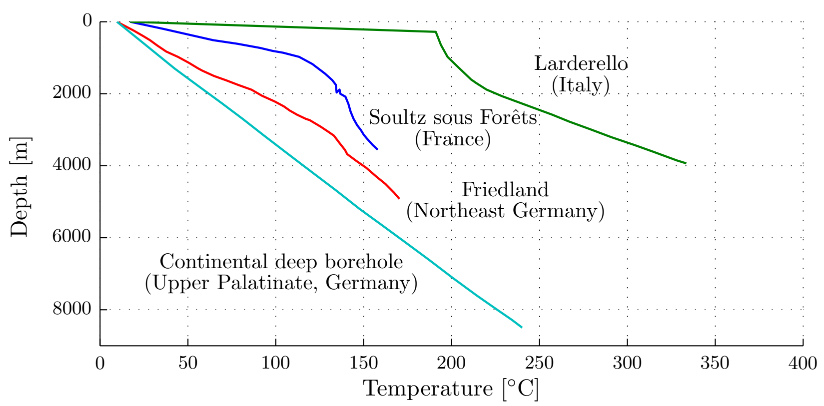


Figure 1: Temperature gradients in different European boreholes for geothermal energy use (Kaltschmitt et al., 2001).

With EGS (also known as Hot Dry Rock, HDR), geothermal reservoirs can be created at almost any location without the need for the presence of natural fluid pathways.

Common geothermal projects use two wells or more. The water is injected at one well and heated by convection in the ground. Depending on the temperature and state of the water from the extraction well, it can either be used directly for heating purposes or indirectly for electricity generation. For the latter, a minimum temperature of 150 °C is required (Dickson and Fanelli, 2003, p.14). In this study, the focus is on the use of geothermal energy for district heating (DH).

Important parameters of geothermal heating systems, with a significant effect on the profitability, include the temperature, flow rate, well depth, and the thermal load density (Piatti et al., 1992, p.82). Temperature and flow rate determine the extractable thermal power. They strongly depend on local ground properties and well depth. The thermal power $\dot{Q}$ of geothermal wells can be calculated as

$$\dot{Q} = \dot{V} \cdot \rho \cdot c_p (T_{out} - T_{in}), \tag{1}$$

where $\dot{V}$ is the flow rate [in $\mathrm{m^3\,s^{-1}}$], ρ the density of the fluid [in $\mathrm{kg\,m^{-3}}$], c_p the thermal capacity of the fluid [in $\mathrm{J\,kg^{-1}\,K^{-1}}$], and T_{out} and T_{in} the well extraction- and re-injection temperatures [in K], respectively. This shows that the power of a geothermal DH system increases linearly with temperature and flow rate as key parameters. Depending on the thermal power produced, more or less households can be supplied with space heating. Thus it is important to determine these key parameters when assessing geothermal projects.

In this study, we focus on temperature and assume that the flow rate is exogenous.

3 Real options analysis

"Investment is the act of incurring an immediate cost in the expectation of future rewards"(Dixit and Pindyck, 1994, p.3). For geothermal projects this means, for example, investing in exploration with the expectation of successfully building a geothermal power plant and eventually selling the generated electricity in the market.

Investment decisions are in most cases characterized by being irreversible, implying uncertainties about future rewards and being flexible in timing (Dixit and Pindyck, 1994, p.3). The investment into a geothermal system, such as drilling a production well, is irreversible since it constitutes a very project-specific investment which cannot be traded on a liquid market. The success of this investment, and therefore any future rewards, is uncertain because the well temperature can only be evaluated to a certain degree prior to drilling. Furthermore, the timing of an investment is flexible and might be more favorable with the prospect of increasing energy prices, or decreasing costs, at some point in the future.

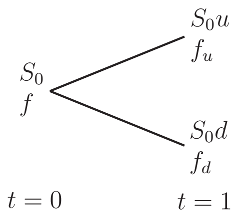


Figure 2: Simple binomial lattice for the option calculation.

A relatively new approach for evaluating investment projects with uncertainties has been introduced with ROA (Dixit and Pindyck, 1994; Schwartz and Trigeorgis, 2001). This valuation technique is based on option pricing methods used in finance and developed by Black, Scholes and Merton (Black and Scholes, 1973; Merton, 1973), who based their valuation on partial differential equations. Cox et al. (1979) (hereafter: CRR) later developed a discrete-time binomial lattice model as an alternative for the valuation of real options that was extended by He (1990) for continuous-time analysis. In the present study, ROA is based on sequential investments with the option to abandon the project in case developments turn out to be unfavorable. The key variable which accounts for future uncertainties in option pricing is the volatility σ. In finance, volatility is estimated by using historical data of an asset for calculating the standard deviation of the logarithmic returns. For real options this is only possible if the success of a project is based on a tradeable asset, such as an energy commodity. In all other cases, volatility either needs to be estimated by experts who are familiar with the project or by simulation of possible future returns (Luehrman, 1998; Mun, 2006).

3.1 Lattice-based options valuation

In this study we apply the option modeling technique based on lattices. We consider a call option value f with strike price B. The strike price is the price for which the underlying can be bought at $t = 1$. In fig. 2 a tree is constructed for an option with price f and an underlying asset with value S_0 at time $t = 0$. In binomial trees, after one time step, only two future developments of the underlying are considered. One is an upward-movement by the factor u, the other one a downward movement by the factor d[1]. Therefore, the possible values of the underlying at time $t = 1$ are S_0u or S_0d. The pay-off from the option f_u or f_d at time $t = 1$ depends on the development of the underlying and the strike price B, on which the option is based.[2]

$$f_1 = max[S_1 - B, 0] \tag{2}$$

[1] u (d) stands for an upward (downward) movement, with $u > 0$, $d < 0$ and $d < 1 + r < u$ (see Cox et al., 1979, p.249).

[2] For example, if the future value turns out to be $S_1 = S_0u$ and the strike price is B, then the pay-off from the option for one share would be $f_u = S_0u - B$.

For this simple model, as indicated in eq. (2), usually the pay-off for an upward-movement is positive and the pay-off for a downward-movement is zero since the option for a negative price development will not be exercised.

In order to calculate the option value at $t = 0$, a portfolio consisting of x shares of the underlying and a short position in one option is constructed. Its value at $t = 1$ is either

$$S_0ux - f_u \tag{3}$$

or

$$S_0dx - f_d. \tag{4}$$

The portfolio is riskless if the value for both developments is equal and the riskless interest rate r is earned from $t = 0$ to $t = 1$. Therefore, the amount of shares can be calculated by setting both cases equal, resulting in x shares of the underlying,

$$x = \frac{f_u - f_d}{S_0u - S_0d}. \tag{5}$$

If we discount the future value of the portfolio to the present, the value is

$$(S_0ux - f_u)e^{-r\Delta t}, \tag{6}$$

which should be equal to the cost for setting up the portfolio

$$S_0x - f = (S_0ux - f_u)e^{-r\Delta t}. \tag{7}$$

By solving for f, substituting from eq. (5) and simplifying, we get

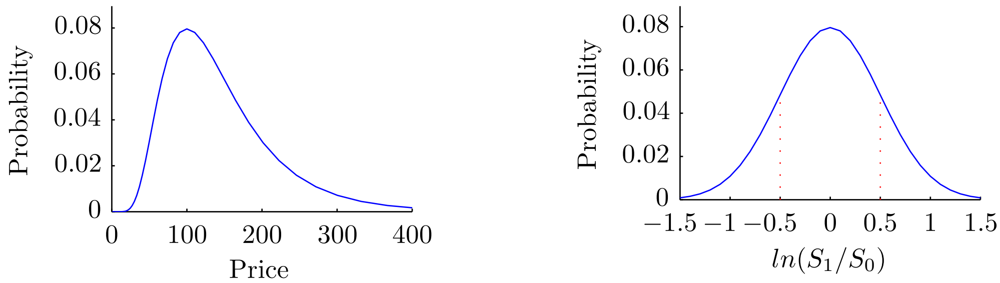
$$f = e^{-r\Delta t}[pf_u + (1 - p)f_d] \tag{8}$$

with

$$p = \frac{e^{r\Delta t} - d}{u - d}. \tag{9}$$

In this equation, p has the characteristic of a probability. It is called the risk-neutral probability, which is not the probability of an upward-movement, but the probability that the return from the underlying will increase by the risk-free interest rate in a risk-neutral world (Hull, 2005, p.245).

Using eqs. (8) and (9) an option value that depends on u, d, r, Δt and B can be calculated. The variables u and d account for the future uncertainty in the development of the underlying. They can be chosen in various ways depending on the characteristics of the underlying. In order to implicate future developments of the underlying into option valuation, the expected probability density function of future underlying values has to be known. Those future values are commonly assumed to be log-normally distributed



(a) Asset prices

(b) Logarithmic returns

Figure 3: Distribution of the price and return of an asset at $t = 1$ for $S_0 = 100$, $\sigma = 0.5$.

and characterized by a volatility measure denoted by σ (Wilmott et al., 1995; Hull, 2005). According to Hull, volatility is defined as the standard deviation σ of the return provided by a financial instrument in one year when the return is expressed using continuous compounding (Hull, 2005, p.286).[3] As an illustration, in fig. 3(a) the distribution of possible underlying values are plotted after one year with volatility $\sigma = 0.5$. The values are log-normally distributed. In contrast, in fig. 3(b) the logarithmic returns at $t = 1$ are normally distributed with standard deviation $\sigma = 0.5$.

The estimation of volatility is the most crucial part for the pricing of options. In finance, this is usually done by analyzing historical data. The simple binomial model, which was constructed earlier on according to fig. 2, can be used to implement the parameter volatility σ into option pricing. This was first done by Cox et al. (1979), who defined the parameters for an upward- or downward-movement u and d as:

$$u = e^{\sigma\sqrt{\Delta t}} \quad and \quad d = \frac{1}{u} = e^{-\sigma\sqrt{\Delta t}}, \tag{10}$$

where the parameters account for the volatility of an underlying asset and the considered time interval Δt.

By using the simple binomial model only two future price developments are assumed for the pricing of an option. In reality, in contrast, the future price can develop in many ways according to the distribution depicted in fig. 3(a) (Wilmott et al., 1995, p.23). The simple binomial model is only a crude assumption which leads to an approximate option value but does not represent the exact option value which can be calculated by continuous analysis using the Black-Scholes-Merton model (cf. He (1990)). In order to calculate the option value using binomial trees and accounting for all possible developments the simple model can be expanded. This is realized by constructing a multi-period tree with n periods and adjusting the parameters u and d according to eq. (10) for each period with time interval Δt. For the calculation of the multi-period option value at time $t = 0$ the tree with possible values of the underlying is constructed first by using u and d. Next, the option

[3]Common values for σ are between 0.15 to 0.6 (Hull, 2005, p.286).

values of the end nodes are calculated with eq. (2). After that, the tree is evaluated by calculating the option value using eq. (8) at each node. This way the option value at $t = 0$ can be calculated. For large values of n this value converges to the value which can be calculated using the formulas of Black-Scholes-Merton.

3.2 RO valuation of oil projects

In this section, we present some applications of ROA, with a particular focus on investments in the oil sector. The reason is that oil investments are similar to geothermal investments, showing many common characteristics. In general, oil projects can be divided into the three stages Exploration (EXP); Development (D); and Extraction (EXT) (Paddock et al., 1988). In the Exploration phase, a potential reserve is examined by performing seismic measurements and drilling activity. The aim is to obtain new information about the reserve itself, which means getting geological data and estimating the quantity of oil that can be extracted. Once there is sufficient potential for the cost-efficient extraction of oil, the development stage is entered. This means platform construction starts and extraction wells are drilled. The undeveloped reserve thereby becomes a developed reserve (Paddock et al., 1988). After the development phase, extraction of oil can start. Here, the primary uncertainty for the amount of future returns is the uncertainty in the price development.

When looking at the different sources of uncertainty for the future success of an oil project, it becomes clear that at the beginning geological uncertainty is a major concern. This means the quantity and quality of oil in the reserve is unknown. As the project proceeds, geological uncertainties are reduced and economic variables become the primary source of uncertainty for the future success of the project. In geothermal investments the situation is very similar. First, geological uncertainties, such as temperature, permeability and structure of the location, have to be investigated. Once these parameters are found to show favorable conditions, the project can proceed and economic variables, such as the prices for heat and electricity, become more important. This analogy is summarized in table 1.

The application of ROA for oil projects was first studied by Paddock et al. (1988). The authors find a discrepancy in the valuation by companies and the government and try to explain it by ROA. Common practice in the oil industry is that governments offer offshore leases in an auction and companies can acquire these. In the valuation process governments always seem to underestimate the value of leases. Paddock et al. (1988), therefore, further investigate the options in oil investments by applying models developed by Brennan and

Table 1: Sources of uncertainty in oil and geothermal investments.

	Oil investments	Geothermal investments
Geological	Quantity and quality	Temperature and flow rate
Economic	Oil price	Price for heat/electricity

Schwartz (1985), who focus on natural resource investments, and McDonald and Siegel (1986), who apply ROA in order to find the optimal time for investment. These models are based on partial differential equations (PDE) developed by Black-Scholes-Merton. The focus of Paddock et al. (1988) is set on the extraction stage. Ekern (1988) study the additional value of flexibility in oil projects by looking at different possibilities of extending extraction. Hurn and Wright (1994) look at irreversible oil production investments in the North Sea, considering uncertainties in price as well as uncertainties in geological conditions. The various modeling techniques and their application in oil investments are presented by Smith and McCardle (1999). The authors use binomial trees for the modeling of various option types in oil projects and optimize them using dynamic programming (DP). The RO model is compared to decision analysis approaches. Cortazar et al. (2001) build up a model by using PDE that is based on the model developed by Brennan and Schwartz (1985). Geological-technical uncertainty is modeled as being high in the beginning of a project and decreasing as exploration proceeds.

An overview of different approaches used for the valuation of oil projects is given by Dias (2004). Specifically, the author compares the modeling of uncertainty in terms of a geometric Brownian motion and mean-reverting processes. Armstrong et al. (2004) combine ROA with Bayesian analysis in order to account for multiple uncertainties in oil investments. A relatively new approach to ROA is proposed by Uçal and Kahraman (2009), using fuzzy logic for the modeling of uncertainties in oil investments. Overseas oil investment decisions are analyzed by Fan and Zhu (2010) also using ROA. The authors compare different countries with a particular focus on China and study the influence of various economic parameters.

An overview of the reviewed literature is given in table 2. All studies focus on the modeling of economic parameters as the primary source of uncertainty and some also include geological uncertainty. When geological uncertainty is modeled, it is not exactly quantified and often relies on the assumptions of experts. A log-normal distribution is also assumed for geological uncertainty, which is characterized by its volatility. In our study, we

Table 2: ROA of oil investments in the literature.

Reference	Uncertainties addressed	Method	Stage
Paddock et al. (1988)	Economic	PDE	EXT
Ekern (1988)	Economic	Binomial	EXT
Hurn and Wright (1994)	Economic + Geological	PDE	EXP D EXT
Smith and McCardle (1999)	Economic + Geological	DP	EXP D EXT
Cortazar et al. (2001)	Economic + Geological	PDE	EXP D EXT
Dias (2004)	Economic + Geological	Binomial	EXP D EXT
Armstrong et al. (2004)	Economic + Geological	PDE	EXP D EXT
Uçal and Kahraman (2009)	Economic + Geological	PDE	EXP D EXT
Fan and Zhu (2010)	Economic	PDE	EXT

quantify uncertainty based on data obtained from the project in The Hague and develop a RO model that is able to account for the character of the temperature uncertainty in geothermal investments.

4 Economic evaluation of geothermal projects

Geothermal projects need to be carefully evaluated before making a major investment. In order to get an idea of the overall project costs, the project is divided into stages and costs are estimated. During the project assessment also risks and uncertainties have to be considered, which may have a significant influence on the project's economic feasibility.

4.1 Cost assessment

Geothermal projects are difficult to evaluate due to the strong influence of local conditions. Furthermore, geothermal energy use is a fairly new and therefore not yet widespread technology. Also, the literature on geothermal project evaluation is still scarce.

Despite of this, it is hard to distinguish between different phases of geothermal projects, as many actions may take place simultaneously. A common approach for breaking down the investment process is to divide it into Identification, Exploration, Drilling, and Production (cf. fig. 4). Numerical modeling of geothermal reservoirs has started to play an increasingly important role in geothermal assessments in recent years, and is capable to decrease uncertainties significantly. Hence it constitutes an additional investment often considered worth making before drilling. Therefore, we have added modeling in the investment process as a distinct additional phase (cf. fig. 4).

Multiple definitions for costs can be found, which vary from absolute total [\$], power-specific [$\$\,\mathrm{kW}^{-1}$] and energy-specific [$\$\,\mathrm{kWh}^{-1}$] to time-dependent values [$\$\,\mathrm{y}^{-1}$]. In addition to different units used in investment calculations, the majority of values found relate to investments in geothermal power plants and not to district heating plants. Nevertheless, the investment phases of identification, exploration and drilling are very similar for both project types. In the following, we present the total investment costs for the different stages of project development. Additional information on costs can be found in appendix A.

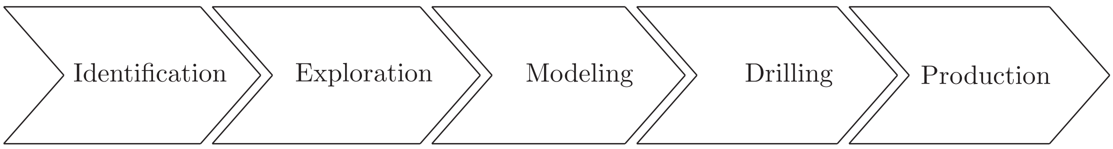


Figure 4: Investment phases for geothermal projects (Source: based on (Deloitte, 2008)).

4.1.1 Identification

Before a geothermal project is started, possible locations have to be identified. Often locations are considered where information about the subsurface already exists, e.g., at sites where exploratory drilling for oil or gas resources has been conducted. This data has to be acquired, since it provides valuable information about the potential of locations. Data relevant for geothermal projects are bottom-hole temperatures (BHT), which were measured in exploratory wells; sometimes temperature logs from exploration can be found as well. In addition to data from drilling, information is gathered from historical geological and geochemistry resource studies. Data from heat flow maps is analyzed, which can be found in special sources such as the Atlas of Geothermal Resources in Europe (Hurter and Schellschmidt, 2003). If the site is found to be promising, land has to be acquired. Cost estimates for this first phase of geothermal projects range from $ 0.385 - 1 million (Deloitte, 2008; Heidinger, 2010; Williamson, 2012) and land acquisition typically takes 3-6 months (Deloitte, 2008).

4.1.2 Exploration

During exploration new information about the site is gathered by performing various kinds of studies. Possible approaches are: (1) initial exploration; (2) geological surveys; (3) geochemical surveys; (4) geophysical surveys; (5) temperature gradient measurements and exploratory wells (Barbier, 2002; Taylor, 2007).

Investments that are made in this phase aim at gaining new information for project evaluation. Costs for this stage can be found in the literature but vary greatly. One reason is that exploration expenses are not strictly defined. They can include various types of surveys that cannot easily be isolated from other stages and which are based on different project conditions. Table 3 gives an overview of these costs. The time frame for exploration strongly depends on local geological conditions and the success of the exploratory surveys. This phase typically takes 1-4 years, with an average value of two years (Sener et al., 2009).

Table 3: Total exploration costs from the literature.

Value	Unit	Source
1.85	M€	Heidinger et al. (2006)
5-10	M$	Deloitte (2008)
1	M€	GEOFAR (2011)
9	M$	Williamson (2012)

4.1.3 Modeling

Techniques of log interpretation and stochastic modeling can help to significantly reduce the uncertainty in the reservoir's temperature distributions (Vogt et al., 2010; Mottaghy et al., 2011). The effort for numerical modeling is small compared to the information that can be obtained. Still, since this method is relatively new, data on cost factors is hard to find in the literature. In our study, costs were estimated at the E.ON ERC/GGE for log interpretation and stochastic modeling after the exploration phase. Main costs are the salaries for scientists and technicians. Furthermore, costs for computational software and hardware need to be added. The overall costs come down to about €100,000. Modeling usually takes from six months up to one year.

4.1.4 Drilling

In the drilling phase, production and re-injection wells are drilled. Costs depend strongly on geological conditions and the targeted depth. In this phase, uncertainties with respect to achievable temperatures are revealed, which account for the success or failure of a project. Well drilling is the main cost that has to be considered at this stage. Costs increase linearly with time, whereas drilling progress decreases with larger depth (Frick et al., 2010). This results in an exponential increase of costs with depth.

There are modeling approaches investigating the relationship between depths and drilling costs. For instance, Tester and Herzog (1990) developed an early model based on an exponential function. This approach was refined through research of Sandia and DOE, which resulted in the so-called WellCost Lite Model (Tester et al., 2006). Klein et al. (2004) present a simple polynomial approach for drilling cost evaluation. Table 4 lists the total values of drilling investments found in the literature. Drilling typically takes about 1-2 years (Deloitte, 2008).

Table 4: Total drilling costs from the literature.

Value	Unit	Source
35 - 50	M$	Deloitte (2008)
10.4	M€	Reif (2008)
2.52	M$	Sener et al. (2009)
5 - 10	M€	GEOFAR (2011)
15	M$	Williamson (2012)

4.1.5 Production

During the production phase, final project investments are made. For electricity generation, this encompasses the construction of the power plant and grid connections. For district heating systems, the main investments are the distribution network, service connections and heat-transfer stations. In both cases, pumps for the geothermal reservoir are needed. Costs that account for the overall investments in the production phase are listed in table 5.

In this study, we consider investments in geothermal district heating. Reif (2008) gives a detailed overview of the main expenditures, which include the components (1) Pumps & accessories (€800,000); (2) Geothermal station & equipment (€2,100,000); (3) Peak-load heating plant (€ 800,000); (4) Distribution network (€ 19,500,000); (5) Service connections (€ 6,600,000); and (6) Heat-transfer stations (€ 5,400,000). Similar cost listings can be found in Erdogmus et al. (2006) for Turkey.

Generally speaking, costs vary greatly and depend strongly on local conditions. Nevertheless, table 5 gives an overview of costs and the order of expenditures that need to be considered in the production phase, which is assumed to take 1-2 years (Deloitte, 2008).

Table 5: Production costs from the literature.

Value	Unit	Source
75 - 85	M$	Deloitte (2008)
35	M€	Reif (2008) (heat)
20	M€	GEOFAR (2011)
60	M$	Williamson (2012)

4.2 Risk and uncertainty

There are many risks and uncertainties associated with geothermal energy projects. Until a well is drilled and the actual temperature can be determined, the actual subsurface conditions are uncertain (Thorsteinsson and Tester, 2010). In order to limit risks and optimize the economic performance of a project, uncertainties should be resolved before making such large irreversible investments. Investments with low probability of success can only be avoided to some extent. Especially at the beginning of a project, the probability for the successful completion of the project at the site in question is low. The probability of successfully completing a project can be increased by investing in identification, attaining information and considering multiple promising sites where data is already available. In the exploration phase, knowledge about the site increases and only at very promising locations drilling will be started. After the first drillings, a better estimation about the potential and future success of a project can be made.

In general, risks and costs have to be evaluated before investments are made. The whole process for geothermal projects is similar to the evaluation of oil or gas projects (Häring, 2007). Economic evaluation methods that account for uncertainties, such as ROA or decision-tree analyses, are applied by many oil companies. These models help improving the economic performance of a company by improved decison-making (Jonkman et al., 2002; Frick et al., 2010).

Thermal power is influenced by parameters such as the subsurface temperature and the established flow rate. In our study, temperature uncertainty is considered as the main source of uncertainty. For now, uncertainty in the flow rate is not modeled. The connection between investments undertaken and the reduction of uncertainty is investigated. In our analysis, data gathered during the evaluation process of a geothermal district heating project in The Hague, The Netherlands is used (for further details on this project see

Mottaghy et al., 2011). In the following, the different phases concerning revelation of temperature uncertainty are explained by using the example of the approaches adopted in the The Hague project.

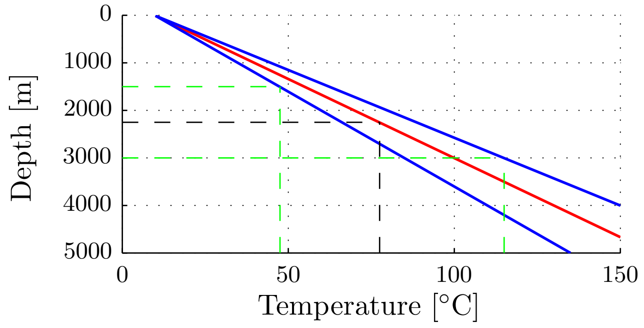
4.2.1 Identification

Before identification starts, temperature uncertainty is high. The actual temperature gradient may differ largely depending on the location concerned (section 2). Before a project is pursued, no information about the local temperature gradient is known. It can be assumed that the average gradient of $30\,\mathrm{K\,km^{-1}}$ can be found, which may vary with a standard deviation of $5\,\mathrm{K\,km^{-1}}$ (see fig. 5(a)). Since the depth of a possible production layer is unknown, the overall uncertainty of the achievable temperature at the location is even higher. The layer is roughly assumed to be located in depths between 1500 m to 3000 m, which results in an uncertainty for obtainable temperatures of about 34 K. The average temperature that can be expected is 77 °C.

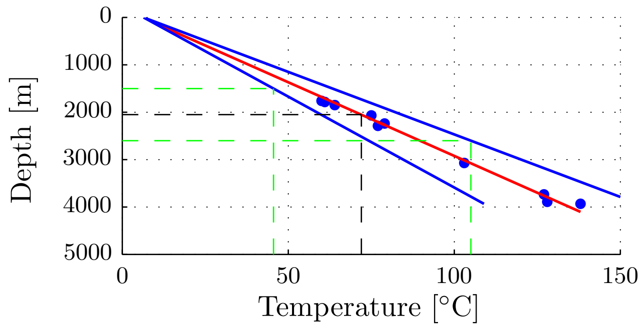
In the The Hague area drillings from oil and gas exploration exist. Data from those drillings was attained in the form of bottom-hole temperatures (BHT) at different locations and depths. Additional information gathered provides evidence on the presence of a sandstone layer (Delft Sandstone), which seems promising for geothermal heat production. Nevertheless, the exact depth of this layer is unknown. In fig. 5(b) the BHTs are plotted according to their depths. For each BHT, the uncertainty of the temperature measurement has to be considered. Deming (1989) states that this is in the order of 9 K per BHT. The resulting uncertainty in the form of two temperature lines is shown as well. The sandstone layer can be estimated from geological maps to lie in a depth of between 1500 m and 2600 m (Simmelink and Vandeweijer, 2008). If we consider the whole range of possible temperature values, this results in an overall uncertainty in the sandstone layer of 30 K, measured in terms of the standard deviation from the expected mean value.

4.2.2 Exploration

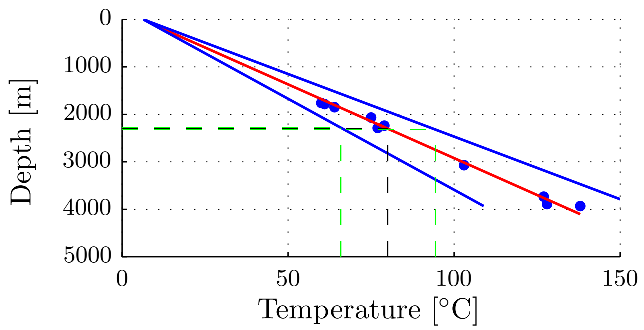
In the exploration phase, different surveys are made to obtain information about the earth's subsurface structure. For the The Hague project especially the results of the seismic surveys are important. The seismic measurements help to narrow down the depth in which the sandstone layer resides. Depending on the depth, the range of possible temperature values varies. In The Hague, the sandstone layer was found to be located at a depth of 2300 m, with a standard deviation of the measurement of 20 m. Thus the temperature uncertainty can be reduced to 14 K in terms of temperature standard deviation. This relation is plotted in fig. 5(c), where the depth of the sandstone layer is determined to $\pm\,20$ m and the upper and lower temperature values can be estimated with less uncertainty.



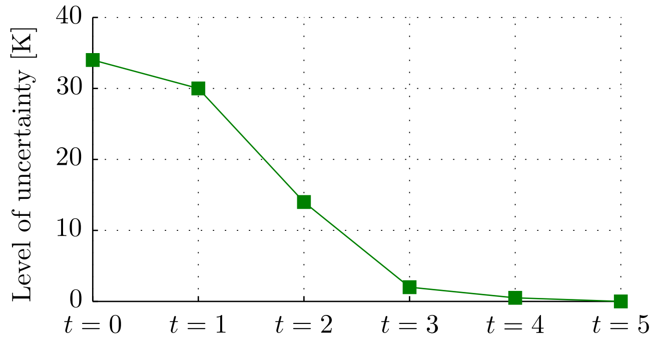
(a) Average gradients and their uncertainty before identification (t=0)



(b) Bottom hole temperatures and the corresponding uncertainties after identification (t=1)



(c) Bottom hole temperatures and the corresponding uncertainties after exploration (t=2)



(d) Development of the uncertainty for the different phases

Figure 5: Uncertainties considered in the different investment phases (($t = 0$) before identification, ($t = 1$) after identification, ($t = 2$) after exploration, ($t = 3$) after modeling, ($t = 4$) after drilling and ($t = 5$) during production).

4.2.3 Modeling

Temperature uncertainty can be reduced further in the modeling phase. Logs and cores from old drilling holes are analyzed and the resulting parameters used in numerical 3-D simulations. Various Monte Carlo simulations are run and the resulting temperature values analyzed statistically. With this method, uncertainty in the reservoir can be lowered to a standard deviation of 2 K. The detailed simulation procedure and results can be found in Vogt et al. (2010) and Mottaghy et al. (2011).

4.2.4 Drilling

In a next step, first holes are drilled at locations which were chosen with the help of 3-D simulations. Temperatures can be measured and first performance tests made. At this stage, the knowledge about the obtainable temperature is very rich. Investments made now are only made under conditions where the success of the project is very likely. In terms of standard deviation, we assume in our analysis that temperature uncertainty after drilling can be reduced to about 0.5 K, which is a conservative assumption.

4.2.5 Production

Investments made in the production phase aim at the completion of the project. Up to this point, uncertainty was reduced stepwise. Now, the probability of success in terms of obtainable temperatures is assumed to be known with certainty. If a sufficiently high operating temperature can be achieved, the project is realized, while otherwise it is abandoned. After the production phase, the operation phase starts. For the The Hague project this means supplying heat to the district heating network. The operation phase is considered as a low-risk project phase, provided long-term contracts guarantee the economic performance. For reasons of simplicity, no risks are assumed at this stage and the success or failure of the project can already be determined in the production phase. Fig. 5(d) gives an overview of the development of the uncertainty parameter over the investment steps. The model used for the evaluation is presented in the following section.

5 Model specification

In our study, a model for the valuation of temperature uncertainties in geothermal projects is constructed. Subsurface temperature is the key variable which accounts for the future success of a geothermal project. Compared to common ROA, where mostly economic parameters are used for option valuation, subsurface temperature is a geological parameter. In the literature, the uncertainty of ground temperatures in geothermal projects is found to be normally distributed (Vogt et al., 2010). Classic ROA typically assumes parameters to be log-normally distributed and characterized by volatility. Since volatility implies a

Table 6: Comparison of commonly applied and geothermal ROA.

	Common ROA	Geothermal ROA
Uncertainty	Economic (e.g. energy prices)	Geological (temperature)
Distribution	Log-normal	Normal
Volatility/Relative sd	Constant	Decreases with time

logarithmic distribution, a new parameter named relative standard deviation σ_{rel} needs to be defined to characterize the normal distribution in eq. (11). It represents the ratio between the standard deviation $sd(S)$ and the mean value S of a normally distributed parameter. Note that, in contrast to the classic volatility definition, it does not assume logarithmic relations.

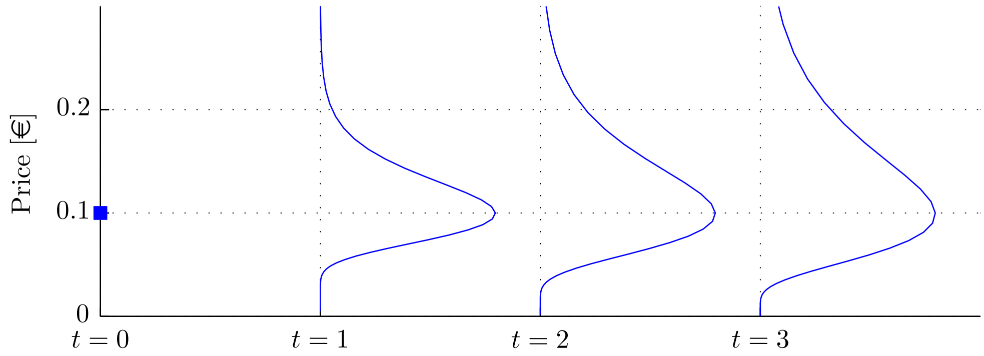
$$\sigma_{rel} = \frac{sd(S)}{S} \tag{11}$$

In comparison to ROA with economic parameters, uncertainty of geological parameters decreases with time (as explained in section 4.2). This relation is plotted in figs. 6(c) and 6(d). In contrast, in fig. 6(a) the temporal development of an economic parameter (price) is shown. At time $t = 0$, the price p_0 is certain and volatility σ is estimated analyzing historical data. For future time steps $t = 1, ..., 3$, distributions for expected price developments are projected based on volatility. The overall uncertainty increases for time steps that lie further in the future. Over the entire time interval, volatility is assumed to be constant (cf. fig. 6(b)).

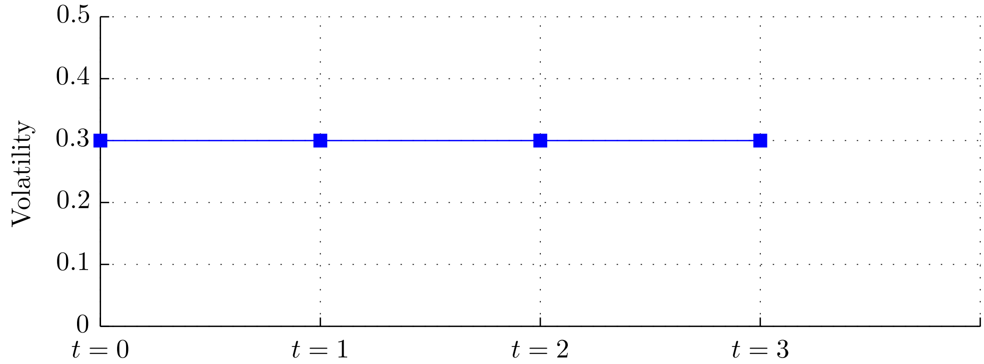
By contrast, only an expected value of temperature T_{exp} with standard deviation $sd(T)$ is known at the beginning of a geothermal project at $t = 0$. Therefore, a probability distribution at $t = 0$ can be constructed, displayed in fig. 6(c). At times $t = 1, ..., 3$, some uncertainties become resolved, which can be observed by a decrease of the relative standard deviation in fig. 6(d). Still, overall uncertainty about the possible actual temperature slightly increases over time, since it reflects distributions based on the large relative standard deviation in the beginning of a project.

The development of future geological values is always based on obtaining new information. Note that this process is not time-continuous, as it is the case with economic variables that are continuously tradeable on markets. Rather, in geothermal projects, new information is obtained by investing in different project phases. The gathering of information is time-dependent since each activity involves a certain time interval. Depending on the duration of each investment step, new information is gained at specific points in time.

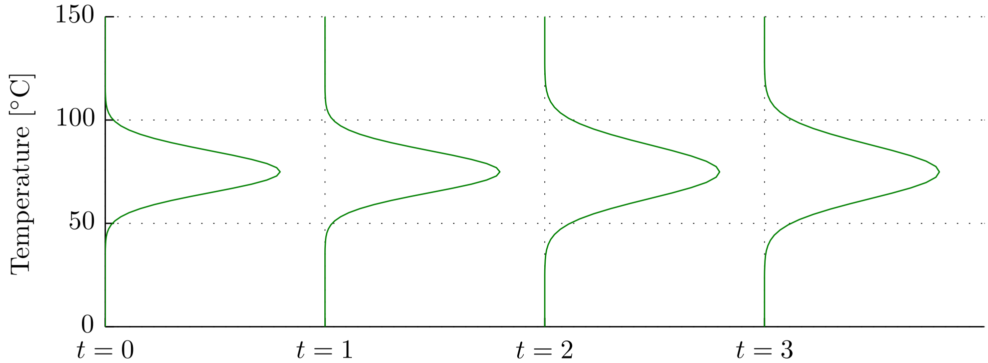
Geothermal projects can be divided into different sequential investments, as explained in section 4. This rather discrete investment process is best modeled using lattice methods (Haahtela, 2010). Lattices are easy to implement and can be applied to value various types of options (Mun, 2006, p.124). Since they are easy to understand, lattices are largely accepted by management, in contrast to the mathematically more sophisticated option-pricing methods using partial differential equations (Mun, 2006).



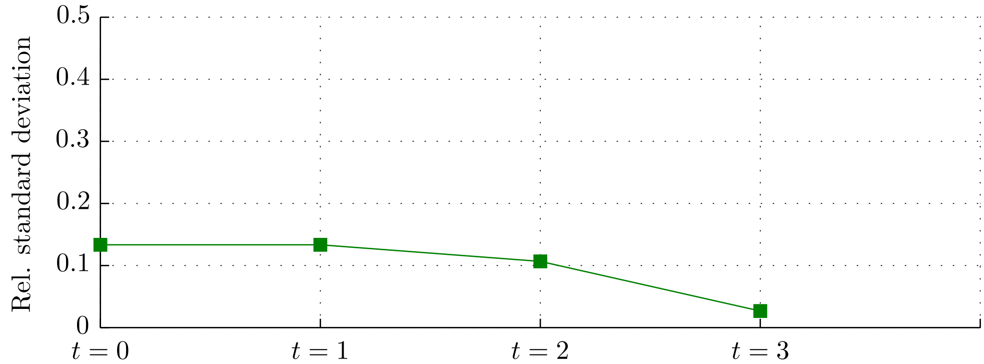
(a) Probability of prices for time steps ($p_0 = 0.1$ €)



(b) Volatility for time steps



(c) Probability of temperatures for time steps ($T_{exp} = 75°$C)



(d) Relative standard deviation of temperature for time steps

Figure 6: Comparison of economic and geological distributions in time.

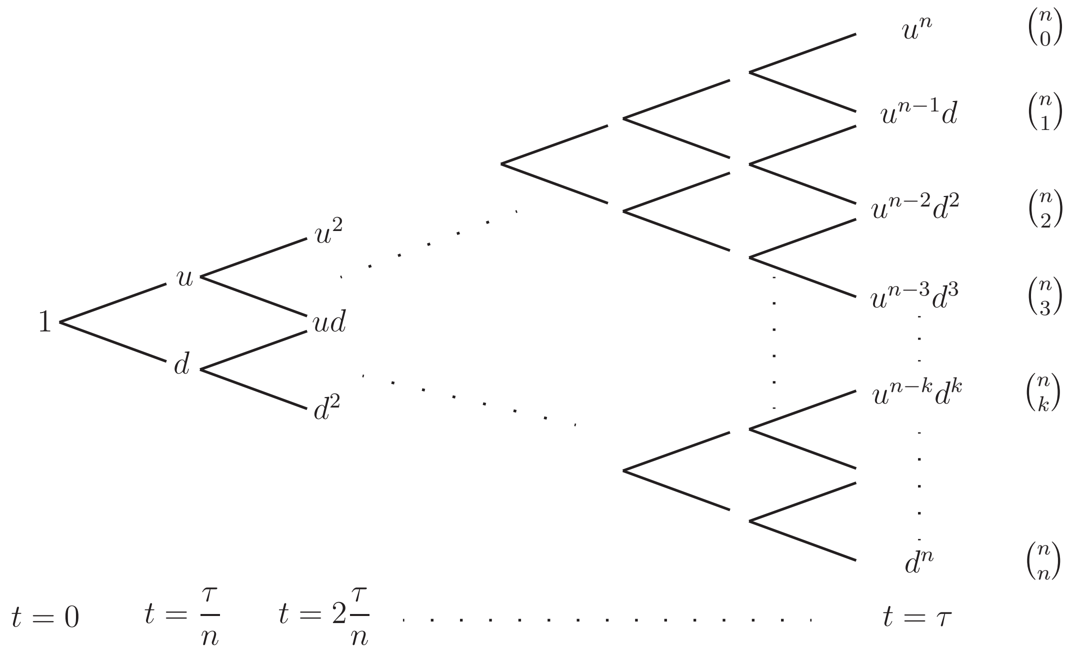


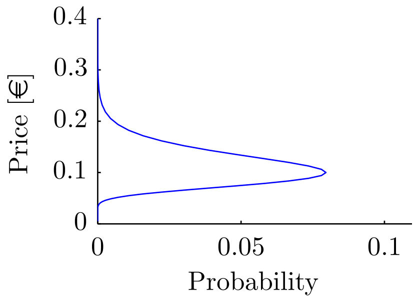
Figure 7: Binomial lattice for time interval τ and n iterations.

The most commonly used lattice in the literature is the binomial lattice developed by Cox et al. (1979). Based on this simple technique new models have been developed, extending the binomial model from trinomial lattices to multinomial lattices (Boyle, 1986; Mun, 2006). Trinomial and multinomial lattices have the advantage of a better resolution of probability distributions in fewer time steps. In addition to the development of multiple lattice trees, different parameters have been applied for the modeling of binomial trees, as summarized by Chance (2007). All of these models are based on the parameter volatility and assume log-normal distributions.

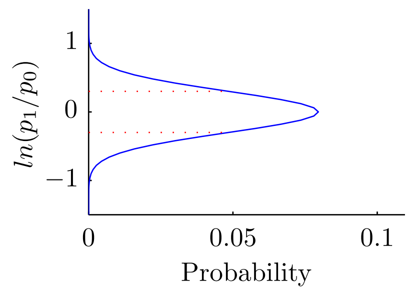
5.1 Normal distributions for binomial lattices

Accounting for normal distributions of parameters, such as subsurface temperature, requires the development of an adequate model.

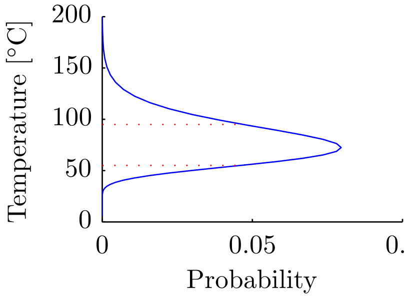
First, the development of common log-normal distributions has to be analyzed to better understand log-normal binomial lattice construction. In fig. 7 the construction of a binomial lattice is displayed for n periods and total time interval τ. The tree starts with 1 and is constructed using parameters u and d for upward- and downward movements. On the right-hand side, possible end node values can be seen, resulting in u^n as maximum and d^n as minimum value. Each node value except for the upper and lower limit can result by following multiple different paths along the binomial lattice. The frequency of occurrence for each end node value can be calculated using binomial coefficients. Depending on frequency and end node values, characteristic parameters of the resulting distribution can be computed. When applying the values u and d proposed by CRR (see eq. (10)), a log-



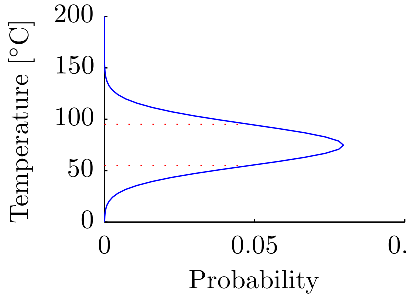
(a) CRR: Probability of price for $p_0 = 0.1$ €, $\sigma = 0.3$ and $t = 1$



(b) CRR: Probability of price for $ln(p_1/p_0)$, $\sigma = 0.3$



(c) Normal: Probability of temperature for $T_0 = 75°\mathrm{C}$, $sd(T) = 20\,\mathrm{K}$ and $t = 1$



(d) Normal, displaced (cf. eq. (19)): Probability of temperature for $T_0 = 75°\mathrm{C}$, $S_\theta = 1000\,\mathrm{K}$, $sd(T) = 20\,\mathrm{K}$ and $t = 1$

Figure 8: Comparison of CRR and normal distribution models

normal distribution is constructed (cf. fig. 8(a)). Characteristic values of this distribution are volatility σ and a logarithmic mean value of zero, which can be computed as follows:

$$\text{log-mean:} \qquad \frac{\sum_{k=0}^{n} \binom{n}{k} ln(u^{n-k}d^k)}{\sum_{k=0}^{n} \binom{n}{k}} = 0 \tag{12}$$

$$\text{log-sd:} \qquad \frac{\sum_{k=0}^{n} \binom{n}{k} ln(u^{n-k}d^k)^2}{\sum_{k=0}^{n} \binom{n}{k}} = \sigma^2. \tag{13}$$

These results can be verified by analyzing figs. 8(a) and 8(b), respectively.

For the ROA of normally distributed parameters, coefficients u and d have to be modified. First, the characteristics of a normal distribution need to be identified. The evaluation has to be based on normal values rather than logarithmic values (see eqs. (12) and (13)). Starting with value 1 in the binomial lattice results in a distribution with a mean value of unity (see eq. (14)). In addition, the relative standard deviation needs to be fitted, which for the binomial lattice displayed in fig. 7 equals the standard deviation. Equations (14) and (15) represent the basic conditions for a normal-distribution-based binomial lattice. Only parameters u and d are unknown in these two equations.

$$\text{mean:} \qquad \frac{\sum_{k=0}^{n} \binom{n}{k} u^{n-k} d^k}{\sum_{k=0}^{n} \binom{n}{k}} = 1 \tag{14}$$

$$\text{sd:} \qquad \frac{\sum_{k=0}^{n} \binom{n}{k} (u^{n-k} d^k)^2}{\sum_{k=0}^{n} \binom{n}{k}} - 1 = \sigma_{rel}^2 \tag{15}$$

For the uncertainties envisaged, the relative standard deviation is based on the duration τ for one investment phase, which means the time interval of the uncertainty parameter and the regarded time interval coincide. Thus, u and d depend only on n as long as they are applied for n periods regarding time interval τ. Equation (14) can be simplified, resulting in the first condition for parameters u and d:

$$u + d = 2. \tag{16}$$

By substituting in eq. (15) and simplifying, we obtain the following results for u and d (for the complete derivation see appendix B):

$$u = 1 + \sqrt{\sqrt[n]{\sigma_{rel}^2 + 1} - 1} \tag{17}$$

$$d = 1 - \sqrt{\sqrt[n]{\sigma_{rel}^2 + 1} - 1}. \tag{18}$$

Binomial lattices built with these parameters result in distributions with the mean as the expected value and a given standard deviation. When using the new parameters u and d for an example of $T_{exp} = 75\,°\mathrm{C}$ and $sd(T) = 20\,\mathrm{K}$, the distribution depicted in fig. 8(c) is obtained. Note that the graph is slightly skewed and thus asymmetric. The reason for this can be found in the general approach of binomial lattices. By using parameters u and d for the construction in multiplicative operations, no negative values can be obtained, which results in skewed distributions. The value of the lower branches always converges to zero.

Haahtela (2010) also observes this characteristic of binomial lattices. To overcome this drawback, a technique named displaced diffusion can be applied when constructing binomial lattices. Displaced diffusion is based on building a lattice with a higher starting value S_θ. The difference between the expected value and the higher starting value is defined as the shifting parameter θ, i. e.

$$\theta = S_0 - S_\theta. \tag{19}$$

In addition, the parameter which accounts for uncertainty, i.e. volatility (in this case measured by the relative standard deviation), is modified. The relative standard deviation is calculated as:

$$\sigma_{rel,\theta} = \frac{sd(S)}{S_\theta} = \frac{sd(S)}{S_0 - \theta}. \tag{20}$$

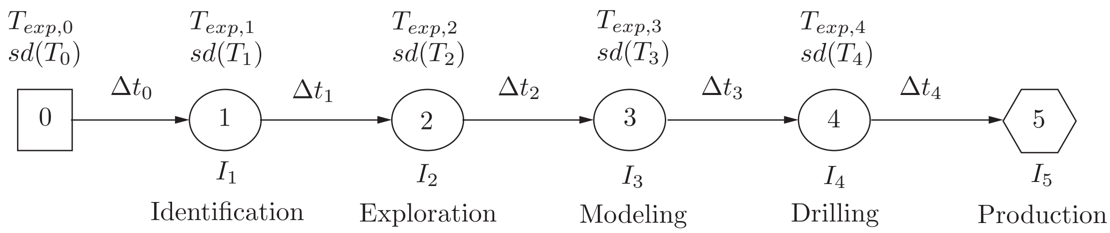


Figure 9: Investment steps for geothermal projects.

After construction of the lattice using $\sigma_{rel,\theta}$ and S_θ, the shifting parameter θ is added to the end node values. This results in a lattice with end node values distributed symmetrically to the expected value S_0, with standard deviation $sd(S)$. By using this technique, also negative distribution values can be accounted for. The graph displayed in fig. 8(d) is built using displaced diffusion. With these parameters, normally-distributed binomial lattices can be constructed that reflect the properties of temperature distributions in geothermal projects.

5.2 Sequential investment with changing level of uncertainty

When investing in the different stages of geothermal projects, the uncertainty of the key parameter temperature is reduced stepwise. After each investment, a new expected temperature $T_{exp,i}$ with standard deviation $sd(T_{exp,i})$ is observed, resulting in a new relative standard deviation $\sigma_{rel,i}$ (cf. fig. 9).

The entire sequential investment process can be modeled by using binomial lattices. After gaining new information at the end of an investment step, decisions can be made on whether to invest in the next phase or to abandon the project. When modeling multiple investment steps, with the arrival of new information, new standard deviations for parameters result, so that the binomial lattices have to be modified. After each investment new up- and downward movement parameters have to be calculated that account for the new level of uncertainty. Since these parameters differ from previous ones, lattice branches do not lead to the same node values and are, therefore, non-recombining. The effort for ROA is increased because simple classic binomial models can only be applied to periods with a constant degree of uncertainty (see Mun, 2006; Hoek and Elliott, 2006).

In order to achieve sufficient accuracy of the option pricing model, each investment step is ideally divided into multiple intermediate steps. These intermediate steps are recombining, since parameters u and d are based on the same relative standard deviation and number of intermediate steps. When modeling the whole process numerically, the simplest approach is to model each branch without considering recombining (Mun, 2006, p.244). This leads to fast growing amounts of data because the number of nodes increases exponentially by the order of 2^{n-1}, with n as the number of overall steps. To reduce the amount of data, binomial lattices can also be modeled as partly recombining during each investment phase with constant relative standard deviation. This decreases the order of data gathered to $2^{i \cdot n_i}$, where i is the number of investments considered and n_i is the number of intermediate steps in each investment stage (Mun, 2006, p.250).

In addition to common binomial lattices, also trinomial lattices are considered for ROA. Trinomial lattices are able to better account for the uncertainties implied in investment processes in fewer time steps. This reduces computational storage needed for the calculations. A basic trinomial lattice is displayed in fig. 14. For the construction an additional parameter m is needed that accounts for the development of the middle branches.

Trinomial lattices were first developed by Boyle (1986). Since then, they were continuously improved and applied for different areas. Recently, Haahtela (2010) introduced a model that is able to construct lattices that account for the modeling of changing volatility and that are still recombining. This makes them numerically very efficient, since the order of operations needed is decreased from exponential to linear dependency by the number of time steps. The whole technique is based on log-normal distributions, with a parameter volatility that does not reflect the distribution observed in geological parameters. We apply this model to geothermal projects in order to compare results based on different approaches and assumptions. A complete description of the algorithm developed by Haahtela (2010) can be found in appendix C.

5.3 Valuing temperature uncertainty

As discussed in section 4.2, temperature uncertainty decreases with each investment step. In order to apply temperature uncertainties to ROA, these have to be transformed into economic values. For geothermal district heating systems, there is a linear dependence between obtainable temperature and system power (see section 2). Dependent on the system power, the amount of heat that can be distributed varies. Usually, heat cannot be distributed any time since loads have to be matched. In other words, a heating plant can only sell heat energy when there is demand by consumers in the district heating network.

The capacity factor cf is the ratio between the amount of heat that is sold to costumers and the amount that could eventually be distributed when the plant operates at full load. For geothermal heating plants, values between 20% and 70% are common (Barbier, 2002).

District heating network tariffs typically are composed of a price both for the power provided at peak times, p_{power}, and for the heat actually delivered to the customers, p_{heat} (2-part tariffs). The power and the amount of heat supplied depend linearly on available temperatures. Temperature uncertainty in the form of standard deviations can, therefore, be directly transformed into economic variables. Equation (21) gives the return for the power provided and eq. (22) for the heat supplied on a yearly basis.

$$R_{power} = \dot{Q} \cdot p_{power} = \dot{V} \cdot \rho \cdot c_p(T_{out} - T_{in}) \cdot p_{power} \tag{21}$$

$$R_{heat} = \dot{Q} \cdot 8760h \cdot cf \cdot p_{heat} = \dot{V} \cdot \rho \cdot c_p(T_{out} - T_{in}) \cdot 8760h \cdot cf \cdot p_{heat} \tag{22}$$

The returns gained during the lifetime of the heating plant can be discounted to the present by calculating the present value of the yearly cash-flows in eq. 23:

$$R_0 = \frac{1 - (1+i)^{-l}}{i} \cdot S. \tag{23}$$

To simplify calculations, constant economic and physical factors are summarized in a factor C_{temp}. This coefficient stands for the linear relation between temperature uncertainty and economic uncertainty. The total returns can thus be written as

$$R = C_{temp} \cdot (T_{out} - T_{in}) \tag{24}$$

with

$$C_{temp} = \dot{V} \cdot \rho \cdot c_p(p_{power} + \cdot 8760h \cdot cf \cdot p_{heat})\frac{1 - (1+i)^{-l}}{i}. \tag{25}$$

The uncertainty in the future returns can, therefore, be expressed as:

$$sd(R) = C_{temp} \cdot sd(T). \tag{26}$$

The economic uncertainty can be applied to ROA, as explained in section 5.1. The shifting parameter θ is not used in the calculations, since high uncertainty values would imply possible temperature values near zero. This does not reflect a physically reasonable distribution. When uncertainty is lower in the latter investment steps, and if the temperature level is sufficiently high, with the developed model the distribution is close to being normally distributed.

5.4 Model parameterization

For the evaluation of the economic feasibility of geothermal plants, it is necessary to determine input parameters that fit the models introduced. In section 4.1, possible investment costs and time frames for investment phases are presented. Note that costs may vary depending on the location and type of geothermal energy use. In table 7 costs are chosen from the values gathered for each investment phase. Depending on the sources, costs are inflation-adjusted and converted to Euros. Values for the time frame and temperature uncertainty are also taken from sections 4.1 and 4.2. Note that the investment volume increases as the project proceeds and the temperature uncertainty decreases. Since all values are estimated and different values can be found in the literature, the robustness of the results needs to be verified by sensitivity analysis.

In addition to parameters for different investment phases, general project parameters need to be chosen. Table 8 summarizes all parameters that are used for the model

Table 7: Input parameters for each investment phase (see section 4 for references).

i	Phase	Costs I_i [M€]	Time frame Δt_i [year]	Temperature uncertainty $sd(T_i)$ [K]
0			0.1	34
1	Identification	0.5	0.3	30
2	Exploration	1	2	14
3	Modeling	0.1	0.7	2
4	Drilling	7.5	1.5	0.5
5	Production	35	1.5	

Table 8: Parameters used for geothermal project valuation.

Category	Subcategory	Symbol	Value	Unit
Economic	Risk-free interest rate[a]	r	0.1	[%] p. a.
	Price for power[b]	p_{power}	2.5	[€ kW^{-1}]
	Price for heat[b]	p_{heat}	0.0537	[€ kWh^{-1}]
	Projected lifetime[c]	l	25	[year]
	Capacity factor[c]	cf	50	[%]
Physical	Expected well temperature	T_{out}	77	[°C]
	Reinjection temperature[d]	T_{in}	40	[°C]
	Volumetric flow rate[d]	$\dot{V}$	42	[L s^{-1}]
	Density of water	ρ	1000	[kg M^{-3}]
	Specific heat capacity of water	c_p	4.18	[kJ kg^{-1} K^{-1}]

[a]Bundesrepublik Deutschland Finanzagentur GmbH (2012)
[b]Geothermie Unterhaching GmbH & Co KG (2011)
[c]IEA (2011)
[d]Mottaghy et al. (2011)

calculations. The risk-free interest rate is derived from German government bonds (Bundesschatzbriefe). Although this interest rate is currently comparatively low, due to the financial crisis, it nevertheless represents the current market situation. In order to relate results to different market situations, other interest rates are applied to the model as well, and changes are observed.

Prices for heat and power were taken from the tariff charged by the geothermal heating plant operator Geothermie Unterhaching GmbH. For geothermal heat plants, the IEA assumes a lifetime of 25 years and a capacity factor of 50% (IEA, 2011). Still, the influence of variations in the capacity factor needs to be observed.

Physical parameters are taken from the geothermal The Hague project, from which data for temperature uncertainty were also available to us (Vogt et al., 2010; Mottaghy et al., 2011). The data reported in the tables summarize the results gathered during the literature review. Note, however, that especially the cost values are only rough estimates. For a concrete project, the local conditions need to be analyzed and costs estimated accordingly. Still, the models presented here can easily be adapted with new input parameters and applied to evaluating geothermal heating plants at any location elsewhere.

Table 9: NPV calculation for an interest rate of 10% p.a.

Phase	Payment implication [M€]	Present value [M€]
Identification	-0.5	-0.5
Exploration	-1	-0.97
Modeling	-0.1	-0.08
Drilling	-7.5	-5.63
Production	-35	-22.79
Operation	15.64	8.83
NPV		-21.15

6 Results

6.1 NPV approach

When calculating the Net Present Value (NPV) of a project, the projected cash-flows are discounted according to their occurrence in time. For geothermal projects, an interest rate of 10% is commonly used for discounting (IEA, 2011). The choice of interest rate can vary depending on the risk preferences and return requirements of the investor and the financial market situation. A sensitivity analysis is carried out to account for different possible values (cf. fig. 10). The investment steps and the associated costs are summarized in table 7. Additionally, the expected return of the project needs to be calculated.

When approaching the geothermal project, a temperature of 77 °C is expected. This results in a projected installed capacity of 6496 kW and an amount of heat energy that is distributed of $28.45 \cdot 10^6$ kWh, assuming a capacity factor of 50% (IEA, 2011). Multiplying these values with the prices presented in table 8 leads to a yearly projected revenue stream of € 1.72 million. Operation and maintenance (O&M) costs for geothermal projects are relatively low and thus neglected in our calculations. When the overall return is discounted over an assumed lifetime of 25 years, this leads to a present value of the revenues of € 15.64 million. In other words, this return can be expected when the project is completed and a temperature of 77 °C has been realized. All cash-flow components need to be discounted to the present and summed up to obtain the NPV of the project (table 9).

When the project is approached as a now-or-never decision the negative NPV of € -21.15 million shows that the project would be a loss-maker and thus should not be realized.

NPV calculations strongly depend on the assumed discount rate. For an exact assessment of the project and its risk, usually a risk-adjusted discount rate is estimated. In order to show the influence of different discount rates on the NPV results, fig. 10 presents the values for different interest rates applied for valuation. As can be seen, the NPV increases for smaller interest rates. The impact of future returns increases for low interest rates since the depreciation of returns decreases. Nevertheless, the NPV is always negative for the project, which means that no favorable market conditions can be expected under which

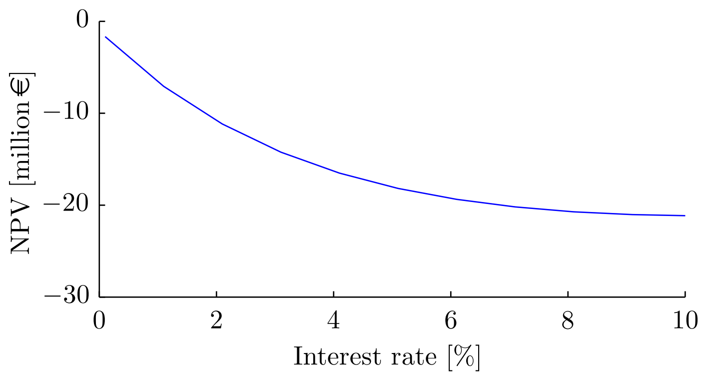


Figure 10: NPV sensitivity for different interest rates.

the project should be realized. Since the geothermal investment means incurring high risks in the form of uncertainties about the successful outcome, a high interest rate is generally applied to account for the risks.

In the NPV approach, the geothermal project is regarded as a now-or-never decision and potential flexibility in the course of project realization is neglected. Project risks are only incorporated in the form of a high risk-adjusted discount rate. The uncertainty in obtainable temperatures is not explicitly modeled. Therefore, the NPV might not be the best-suited decision variable for geothermal projects. Since geothermal projects cannot be pre-determined at the beginning of a project, and since different developments can occur during realization, this should also be implemented into the evaluation process. The rather static NPV criterion is unable to reflect the kind of uncertainty and flexibility associated with geothermal projects, which is why in the present study ROA is applied.

6.2 Real options analysis

In the following, we first present the results for normally distributed uncertainties (section 6.2.1), followed by those from assuming a log-normal distribution (section 6.2.2).

6.2.1 Binomial lattice model (normal distribution of temperature)

The binomial lattice model developed in section 5 is now applied to the evaluation of geothermal projects, using the parameters presented in section 5.4. Multiple investment phases are considered, as presented in fig. 9. The initial phase is modeled in order to account for the uncertainty in temperature that is already present before a project starts (cf. fig. 6). Each of the phases can be modeled in one single step or in multiple steps n_i. The simplest model with only one binomial lattice step per investment phase does not reflect sufficient possible developments, especially for the initial phase, since only two possible developments are considered. It is, therefore, necessary to divide each phase into n_i different steps in order to increase the accuracy of the ROA.

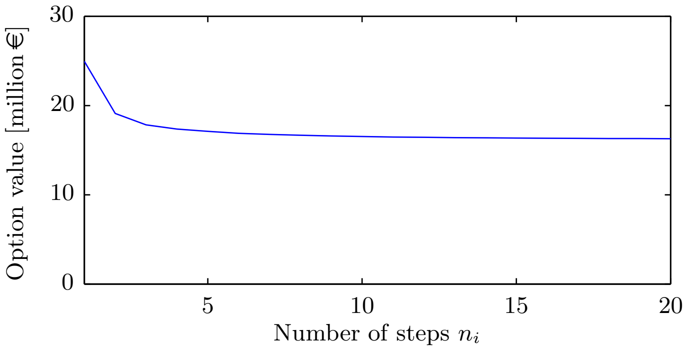


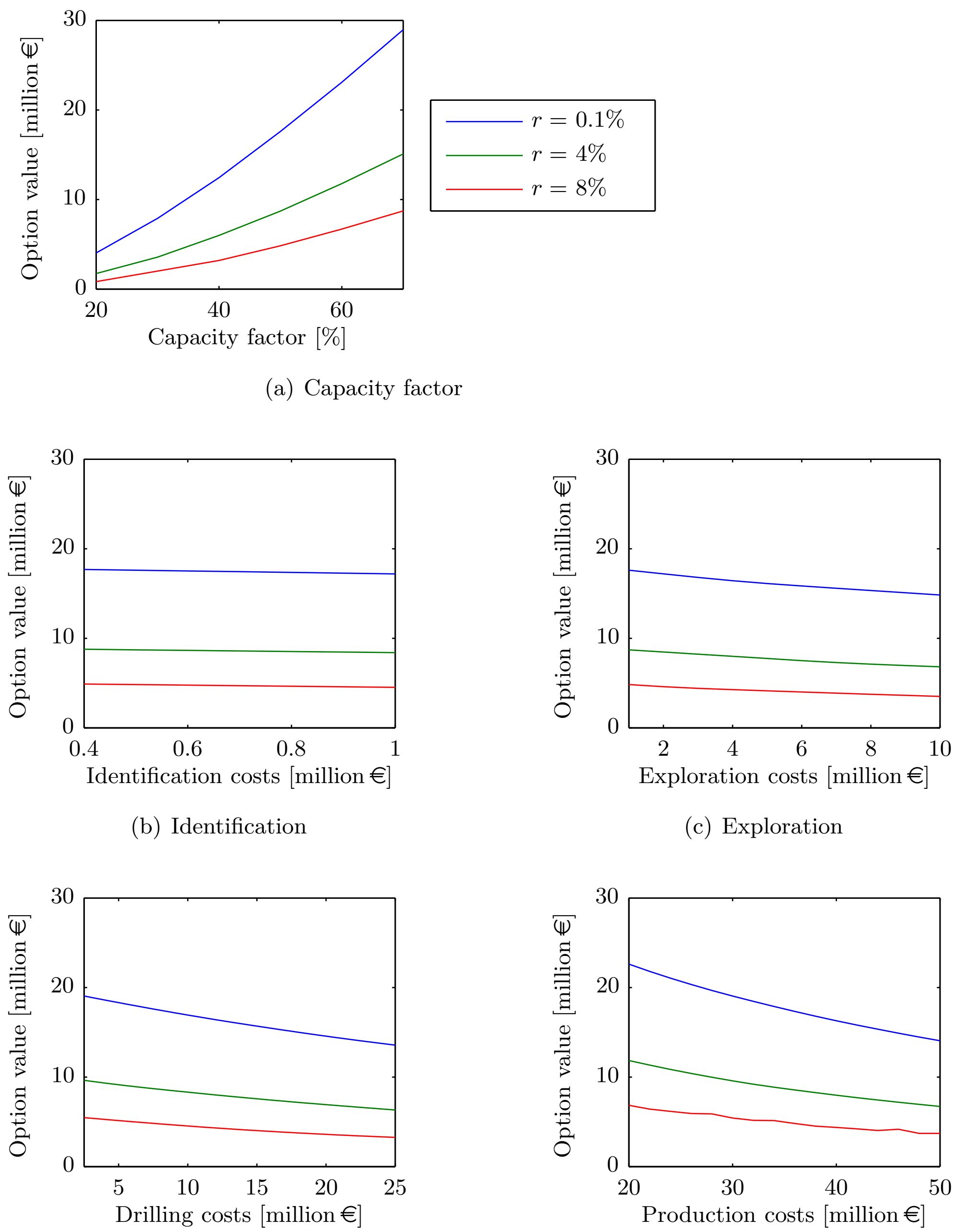
Figure 11: Option value for different numbers of calculation steps

The effect of an increasing number of steps is displayed in fig. 11. For the simple model using the one-step approach, an option value of € 25 million is calculated. The calculation with multiple steps of $n_i = 20$ leads to a more accurate option value of € 16.3 million. As can be seen, increasing the number of steps to more than about 3-5 does not increase accuracy much further.

Both option values are positive, which means the geothermal project should be realized. However, it does not mean that the project will be successful, but that the option on a successful project development has a positive value. By investing, an option is acquired to further proceed with the project if conditions turn out to be promising. Therefore, the initial step of investing into the Identification phase should be undertaken. After the Identification phase, one can proceed with the project if temperatures are expected to be sufficiently high for the successful operation. Otherwise, the project should be abandoned.

The step-by-step revelation of information and the option to stop project development after each phase add an extra value to the project that can be defined as the value of flexibility. The project risk, in the form of temperature uncertainty, is not only seen as a threshold to the project but it can also lead to higher returns when temperatures exceed expectations. It is, therefore, economically reasonable to invest in information revelation before making decisions about the final investment in the form of heat plant construction. As discussed in section 5.4, all input parameters are estimates and results should be verified considering different cost developments as well as market conditions.

Fig. 12 shows the development of the project's option value for different parameters. All results are obtained by modeling $n_i = 10$ intermediate steps for each investment phase. For every development, interest rates of 0.1%, 4% and 8% are considered. As mentioned in section 5.4, the low interest rate of 0.1% for German bonds is also a result of the financial crisis. Generally, higher values would be expected for a risk-free interest rate. Therefore, higher interest rates are also considered when analyzing the influence of each parameter. In general, higher interest rates result in a lower option value, as it was also observed for the NPV calculations (cf. fig. 10). The capacity factor has a strong influence on the feasibility of the project, as can be seen in fig. 12(a). A low capacity factor means that



(a) Capacity factor

(b) Identification

(c) Exploration

(d) Drilling

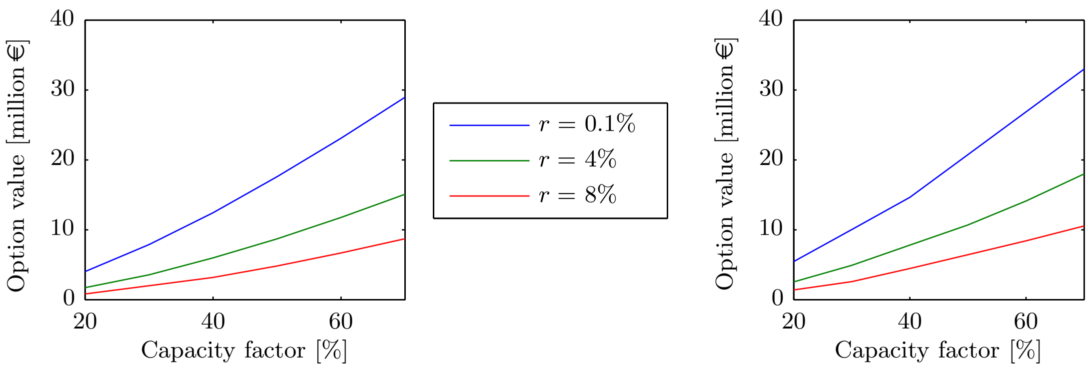
(e) Production

Figure 12: Comparison of different parameter variations.

only a small fraction of the theoretically distributable power can actually be sold in the market. Therefore, the yearly return decreases. Even for small capacity factors and an interest rate of 8%, positive option values result. In contrast, the influence of identification and exploration costs on the option value is relatively small. These investments aim at information-revealing in the beginning of a project, where expected costs are still relatively low. Interestingly, drilling and production costs have a higher impact on the option value. The reason can be found in the relatively high amount of money that needs to be spent during these project phases. Different levels of costs for the modeling phase were not investigated, since variations are relatively small and the costs are very low compared to the other phases. Positive option values are calculated for all parameters investigated. This means that investments should generally be undertaken. After each phase, investors can decide whether or not to proceed with the project.

6.2.2 Trinomial lattice model (log-normal distribution of temperature)

The model of Haahtela (2010) is also used for option valuation. It accounts for three possible developments of the uncertainty parameter at each valuation step. As explained in section 5.2, option values can be calculated with higher accuracy in fewer steps. For the base case, parameters from table 7 are applied and an option value of €20.8 million is obtained, which is higher than the €16.3 million calculated in section 6.2.1 when assuming a normal distribution of temperature. Fig. 13 also shows that the evaluation based on log-normal distributions results in slightly higher option values. The log-normal model does not fit well with the characteristics of temperature uncertainty. The calculated mean value of the log-normal distribution is higher than the actual normally-distributed uncertainty. This shows that distributions always need to be analyzed in detail before assumptions for the modeling are made. Otherwise, option values obtained might be biased and do not match well with the kind of uncertainty characteristics of the considered project.



(a) Binomial normal

(b) Trinomial log-normal

Figure 13: Comparison of results from normally and log-normally distributed models.

7 Conclusion and outlook

In this paper, we evaluated geothermal projects for district heating by applying different investment evaluation techniques. The projects considered were divided into characteristic phases of uncertainty. Both the Net Present Value (NPV) criterion and Real Options analysis (ROA) were applied. Moreover, temperature uncertainty in geothermal projects was modeled as a driving variable in ROA. Since the level of uncertainty changes during the project realization, models that account for changing uncertainty have to be applied.

To this end, a new binomial lattice model was developed for the evaluation of normally distributed parameters. It was modeled as being partly recombining for periods with constant uncertainty. Moreover, a trinomial lattice model based on the log-normal distribution of temperature was used for comparison.

ROA is able to account for the flexibility associated with geothermal projects in the form of a step-by-step investment with revealing uncertainty. When applying the NPV criterion, negative values are obtained, which implies that the project should not be realized. For the ROA, values turn out to be positive, which means investment into the first phase of a project should be effected. After each project phase, decisions about further investments can be made, depending on the expected temperature. When assuming a log-normal distribution, option values turn out to be higher than with normally distributed parameters. The robustness of the results are checked by applying sensitivity analysis. For the geothermal project considered, ROA leads to positive option values in all cases.

Results show that geothermal projects should be evaluated carefully and ROA are appealing for modeling uncertainty. The step-by-step revelation of temperature uncertainty, as well as the option to abandon the project at any stage, add an extra value to the investment. This reduces the investment threshold that is generally associated with geothermal projects. The high level of uncertainty in the beginning of geothermal investment decisions is modeled and incorporated into the valuation process.

In our study, temperature uncertainty is considered as the key parameter to thermal power. Another influence on thermal power is the flow rate, which depends on rock properties such as permeability. For further investigations and in order to model all aspects of geothermal project uncertainty, this parameter should be included into option models as well.

After construction of the plant, there is still some uncertainty present in geothermal projects that was not modeled in our study. Depending on the subsurface structure, thermal breakthroughs may occur after some time. Such an incident leads to a rapid decline in power and limits the projected lifetime. When assessing investments into geothermal projects, the risk of a thermal breakthrough should, therefore, also be accounted for, providing scope for future research.

The model presented in this paper can easily be adapted to evaluate different applications of geothermal energy use. In particular, geothermal electricity generation projects could also be analyzed by applying ROA. Only the relation between temperatures obtained and the resulting power gained needs to be adjusted, which could be implemented in the form of efficiency curves. The assumption of log-normal-distributed parameters in ROA holds for financial assets but is not applicable to all kinds of uncertainty. Thus, the binomial lattice model developed can also be applied to other applications of ROA, provided normally distributed uncertainty is present.

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Appendix

A Review of the literature on costs for geothermal projects

Table 10: Exploration costs from the literature.

Category	Value	Unit	Source
Total	1.85	M€	Heidinger et al. (2006)
	5-10	M$	Deloitte (2008)
	1	M€	GEOFAR (2011)
	9	M$	Williamson (2012)
Power-specific (electric)	150	$/kW	Hance (2005)
	150	$/kW	Kagel (2006)
	183	$/kW	Cross and Freeman (2009)[a]
	127.1	$/kW	Sener et al. (2009)
Levelized (electric)	3.7	$/MWh	Sener et al. (2009)
Time-dependent	0.12	M$/y (var)	Juul-Dam and Dunlap (1975)
	0.2	M$ (fix)	found in Sener and van Dorp (2005)

[a]Exploration with Exploratory Drilling

Table 11: Drilling costs from the literature.

Category	Value	Unit	Source
Total	35 - 50	M$	Deloitte (2008)
	10.4	M€	Reif (2008)
	2.52	M$	Sener et al. (2009)
	5 - 10	M€	GEOFAR (2011)
	15	M$	Williamson (2012)
Power-specific (electric)	750	$/kW	Kagel (2006)
	1367	$/kW	Cross and Freeman (2009)
Levelized (electric)	14.9	$/MWh	Sener et al. (2009)

Table 12: Production costs from the literature.

Category	Value	Unit	Source
Total	75 - 85	M$	Deloitte (2008)
	35	M€	Reif (2008) (heat)
	20	M€	GEOFAR (2011)
	60	M$	Williamson (2012)
Power-specific (electric)	1850	$/kW	Kagel (2006)
	2100	$/kW	Cross and Freeman (2009)
Levelized (electric)	35.4	$/MWh	Sener et al. (2009)
Levelized (heat)	80.5	$/MWh	Sener et al. (2009)(district heating)

B Derivation of coefficients u and d for normal distribution

The first condition is:

$$\mu = \frac{\sum_{k=0}^{n} \binom{n}{k} u^{n-k} d^k}{\sum_{k=0}^{n} \binom{n}{k}} = 1 \tag{B.1}$$

which can be written as

$$\sum_{k=0}^{n} \binom{n}{k} u^{n-k} d^k = \sum_{k=0}^{n} \binom{n}{k} = 2^n \tag{B.2}$$

$$(u+d)^n = 2^n \tag{B.3}$$

$$u + d = 2 \tag{B.4}$$

The second condition states that

$$\sigma_{rel}^2 = \frac{\sum_{k=0}^{n} \binom{n}{k} (u^{n-k} d^k)^2}{\sum_{k=0}^{n} \binom{n}{k}} - 1 \tag{B.5}$$

$$\sum_{k=0}^{n} \binom{n}{k} (u^{n-k} d^k)^2 = (\sigma^2 + 1) \sum_{k=0}^{n} \binom{n}{k} \tag{B.6}$$

$$(u^2 + d^2)^n = (\sigma^2 + 1) 2^n \tag{B.7}$$

$$u^2 + d^2 = 2 \sqrt[n]{(\sigma^2 + 1)} \tag{B.8}$$

$$u^2 + (2 - u)^2 = 2 \sqrt[n]{(\sigma^2 + 1)} \tag{B.9}$$

$$\begin{aligned} u^2 + 4 - 4u + u^2 &= 2 \sqrt[n]{(\sigma^2 + 1)} \\ u^2 - 2u + 2 - \sqrt[n]{(\sigma^2 + 1)} &= 0 \end{aligned} \tag{B.10}$$

Solving the quadratic equation leads to

$$u = 1 + \sqrt{\sqrt[n]{\sigma_{rel}^2 + 1} - 1} \tag{B.11}$$

$$d = 1 - \sqrt{\sqrt[n]{\sigma_{rel}^2 + 1} - 1}. \tag{B.12}$$

C Trinomial lattice parameters

The calculation is based on the algorithm published by Haahtela (2010). First the volatility at each phase is calculated according to eq. (C.13).

$$\sigma_i = \sqrt{\frac{ln\left(\frac{sd(S)_i}{S_0 \cdot e^{rt}}\right)^2 + 1}{t_i}} \tag{C.13}$$

The coefficients for the branches are calculated as follows:

$$\begin{aligned} u &= e^{r \cdot \Delta t + \sqrt{e^{(\lambda \cdot \sigma)^2 \cdot \Delta t} - 1)}} \\ m &= e^{r \cdot \Delta t} \\ d &= e^{r \cdot \Delta t - \sqrt{e^{(\lambda \cdot \sigma)^2 \cdot \Delta t} - 1)}}. \end{aligned} \tag{C.14}$$

The dispersion parameter λ is set to 1.12 as proposed by Haahtela (2010). With coefficients u, m and d the risk-neutral probabilities can be calculated for each branch:

$$\begin{aligned} p_u^i &= \frac{m^2 \cdot (e^{\sigma^2 \cdot \Delta t} - 1)}{(u^2 + md - um - ud)} \\ p_u^i &= p_u \frac{m - u}{d - m} \\ p_u^i &= 1 - p_u - p_d. \end{aligned} \tag{C.15}$$

The trinomial lattice can be constructed using these parameters based on the initial value S_0.

After the construction of the lattice, end node values can be calculated and the tree can be evaluated starting at the end node values. For each step, branch values can be calculated as follows:

$$S_{t-1} = \frac{p_u S_{t,u} + p_m S_{t,m} + p_d S_{t,d}}{e^{r \Delta t}}. \tag{C.16}$$

This way the final option value can be calculated (for more detailed information see Haahtela (2010)).

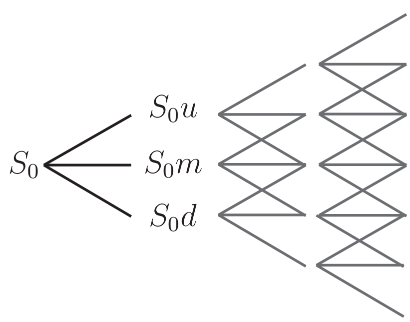

Figure 14: Simple trinomial lattice for option valuation.

List of FCN Working Papers

2012

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