OSCOGEN

Optimisation of Cogeneration Systems in a Competitive Market Environment

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Market analysis and tool for electricity trading

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1 Introduction

The liberalisation of the energy markets will increase the amounts of exchange-based trading of energy. Utilities must be in a position to submit bid functions for selling and buying energy. Consequently, they have to find prices at which they want to sell certain amounts of energy.

In this report the trading structures of different EU markets are first described. Then the requirements and possible realisation of a tool for electricity trading is described. Three different approaches are presented. The first based on the use of a deterministic optimisation model while the second approach includes the possibilities to include uncertainty factors by using stochastic optimisation. The first approach may be a suitable approach for long-term planning while the second approach is developed for short-term bidding, for example placing bids at the one day ahead market, where the uncertainty factors can become very important. The third approach is based on the use of a real option optimisation model. This approach also take the uncertainty factors into consideration.

2 Trading Structures in European Exchange-Based Electricity Spot Markets

Written by Reinhard Madlener and Markus Kaufmann, CEPE

2.1 Introduction

This section provides a synopsis of the prevailing trading structures (mechanisms and traded products) that are currently in place, or under development, at the major exchange-based European electricity spot markets (i.e. APX – The Netherlands, APX UK / UKPX – United Kingdom, Borzen – Slovenia, EEX – Germany, EXAA – Austria, GME – Italy, Nord Pool – Scandinavia, OMEL – Spain, and Powernext – France). The German power exchange EEX in Leipzig (merged EEX Frankfurt/LPX Leipzig) will be discussed in some more detail than the others, because it is of a particular interest for the development of a realistic spot market bidding tool within the OSCOGEN project. For a more detailed coverage of what is being discussed in this section see Deliverable 5.1a /Madlener et al. 2002/.

At a more general level, this section constitutes a guide to better understand the opportunities for generators to place their bids on the various power market exchanges in Europe. It can also serve as an aid in the design of tools for the generation of optimal bids, given certain plant characteristics and a particular market design and situation. Note that although some systems allow trade in spot and future markets, our focus here is restricted to spot markets.

2.2 Basic Aspects Regarding the Trading Structure of Power Exchanges

2.2.1 Markets

On a liberalised electricity market, the participants can act on a variety of markets. Traditionally they can trade electricity bilaterally on the *over-the-counter (OTC) market*, where the bulk of transactions is still being settled. As an alternative in some countries *organised markets, i.e. exchanges*, have been established. Electricity exchanges usually provide at least a *day-ahead market*, where electricity is traded a day in advance, and an operating schedule developed by the system operator. Because of the unavoidable demand-supply discrepancies between the settling of contracts on the day-ahead market and actual physical delivery the day after, exchanges sometimes offer an *intra-day market*, also referred to as *hour-ahead or adjustment market*. This market closes a few hours before the actual physical delivery of the electricity contracted for. Additionally, in order to balance power generation to load at any time during real-time operations, system operators use a *balancing or real-time market*, where participants can submit bids that specify the prices they require (offer) to increase (decrease) their generation, or decrease (increase) their consumption, for a specific volume immediately. Additional *balancing services (*also referred to as *ancillary services)* needed to support a reliable delivery of electric energy (e.g. load following capability, congestion management, transmission losses, reactive power support, among others) are sometimes also traded on an exchange-based market.

2.2.2 Trading System

European exchanges normally provide *bidding-based trading* in contracts for power delivery during a particular hour of the next day (except in England & Wales, where half-hour contracts are traded). The usual trading system is a daily *double-side auction* for every hour to match transactions at a single price and a fixed point in time. Again the UK market is an exception in this respect since trading only takes the form of *continuous trading* (see 2.2.3.2).
In either form participants, by submitting bids, determine how much they are prepared to sell or buy at what prices. Sometimes the possible price values are bounded by a top limit (e.g. EEX in hourly auctions, Powernext). Another special feature to be aware of are limits to price volatility in order to achieve price continuity (e.g. EEX in continuous trading, Borzen). If the potential execution price lies outside these limits, participants are allowed to change their bids in an extended call phase of an auction, or an auction is initiated in continuous trading to get a new reference price.

Usually the participants can add to their bids several execution conditions, and they can offer or ask the same quantity of power for a period of consecutive hours called *block bids*.

2.2.3.1 Auction trading

Figure 2.1 depicts the *basic structure of an auction*. All the submitted bids are collected in a sealed order book, i.e. the participants know only their own bids and cannot see the others. The bids are changeable until the closure of the call phase. For price determination in an auction all the bids collected up to the predetermined closure of the call phase are sorted according to the price and aggregated to get a *market demand and supply curve* for every hour.

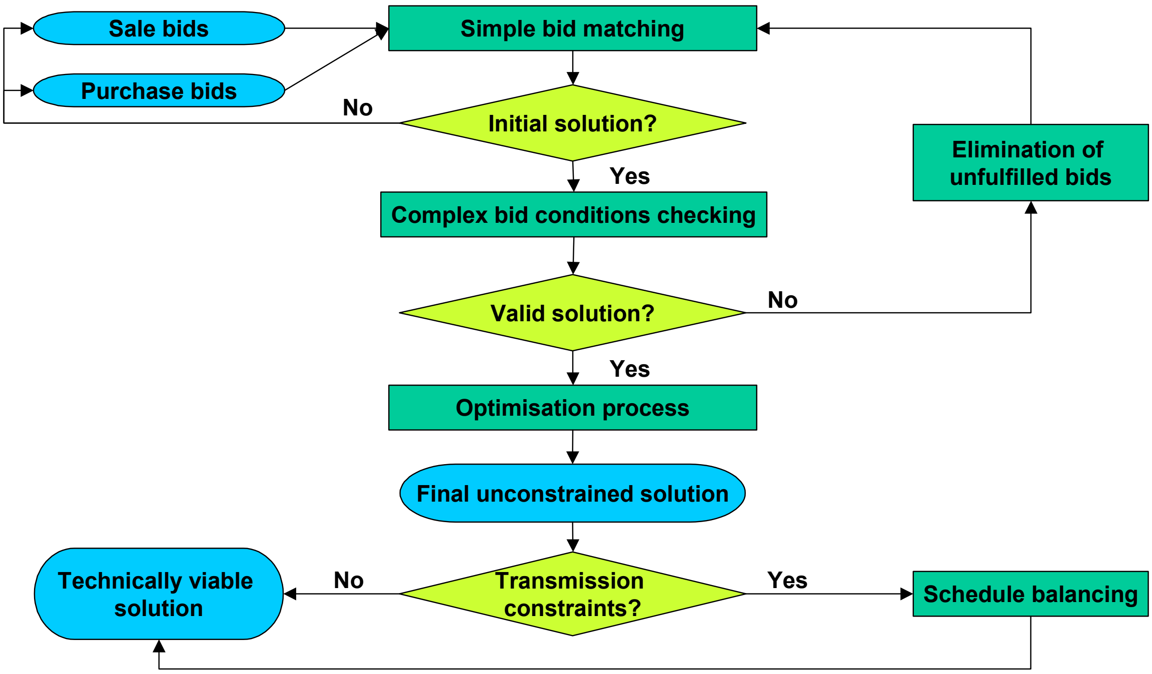

Figure 2.1: Basic structure of an auction

Some exchanges include the *block bids* in the aggregation by changing the blocks into price-independent bids for the hours concerned (e.g. APX, EEX in hourly auctions, Nord Pool). Others use continuous trading to settle block contracts, which will be described below (see section 2.2.3.2).

The simple bid matching ignores any execution conditions or grid capacity constraints and results in an *initial market clearing price* (or *auction price*) for every hour and trade volumes for every bid (see Figure 2.2). The market clearing price is the price level at the intersection of the aggregated demand and supply curves and maximises the trade volume. If no linear interpolation is used to derive the curves, additional pricing rules are applied in case of multiple price levels at the intersection of the two curves. Non-existence of an intersection may trigger a second round of submitting bids. Alternatively, the last calculated market clearing price of the product in question – referred to as the *reference price* – may be used.

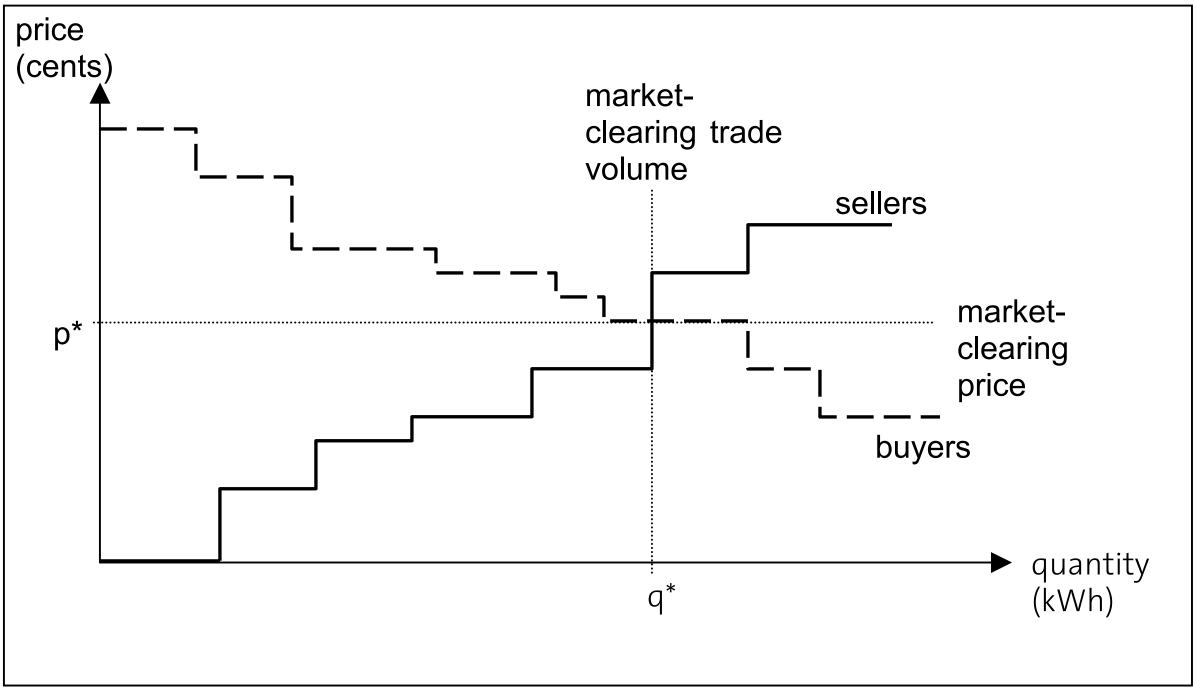

Figure 2.2: Simple bid matching

There may be a *surplus* at the market clearing price resulting of the simple bid matching (e.g. there is a demand surplus in Figure 2.2). In this case the volumes of bids with the market clearing price as limit are proportionally curtailed, or the algorithm selects the bids according to the time of order book entering (first come, first serve).

The *initial solution* has then to be checked against all the *conditions added to the bid*. For a block bid, an average of the market clearing prices for the hours included in the bid is calculated. This price has to be equal or better than the price limit stated by the participant to satisfy the bid (*minimum income* (sales) or *maximum payment* (purchases) *condition*).[1]

If not all conditions are satisfied the price solution is not valid. In this case one of the unfulfilled bids is eliminated and the price calculation is run again. This checking process is iterated until all the remaining bids can be fulfilled.

In some cases (e.g. APX, OMEL) the valid price solution resulting of the bid conditions checking is optimised in a next step. This process tries to minimise the amount of money that removed bids would earn if they were not removed.

[1] When entering a (sales) block bid, the participant defines a block of consecutive hours, a volume applicable for all hours, and a price. The minimum income condition refers to the equation of the number of consecutive hours, the volume, and the limiting price. A block bid can be matched in case the limiting price is equal to, or lower than, the average price throughout the defined block of hours. A block bid must be matched for the entire volume specified, and for all hours. If this is not possible, the block bid is rejected (cf. www.apx.nl/marketresults/aggcurve/disclaimer.html).

The trade volumes of the matched bids have also to be checked against the transmission grid capacities. If there are *transmission constraints*, the schedules have to be balanced either by only adjusting the trade volumes (e.g. OMEL), by adjusting the trade volumes and re-running the iterative bid matching (e.g. APX), or by splitting the market in several areas (e.g. EXAA, GME, EEX, and Nord Pool). This takes place either before (e.g. APX) or after the optimisation (e.g. OMEL) process and results in a *technically viable solution*.

2.2.3.2 Continuous trading

Some exchanges provide an alternative trading form to the auction system called *continuous trading*. This form is used either to only trade block contracts (e.g. Borzen, EEX) or both individual hours and block contracts (e.g. UKPX, APX UK).

Continuous trading differs from auctions in the following ways. First, participants have access to the order book. Second, each incoming bid is immediately checked and matched if possible according to price/time priority. Finally, the *contract price* is not the same for all transactions, as it is determined according to only the concerned bids (pay-your-bid pricing at e.g. APX, UKPX, APX UK) or the order book at the time of the bid matching (e.g. Borzen, EEX). At some exchanges (e.g. Borzen, EEX) continuous trading is preceded by an opening auction and followed by a closing auction. Both auctions are similar to the auction described before.

2.3 EEX

The *German power exchanges in Leipzig* (*LPX*, operational since Jun 2000, 80 participants as of March 2002) *and Frankfurt* (*EEX*, operational since Aug 2000, 60 participants as of Jan 2002), respectively, are currently undergoing a period of transition, since the announcement has been made in October 2001 that the two exchanges will be *merged*. The new exchange, named *European Electricity Exchange (EEX)* and located in Leipzig, will offer its spot market participants a day-ahead auction market as well as continuous trading. The system of the auction market corresponds more or less to the trading system that existed at LPX, while the system of continuous trading, in contrast, is similar to the one that was in operation at the former EEX.

2.3.1 Auction market

On the EEX auction market (weekdays only), *hourly contracts* for every hour of the next day (minimum delivery of 0.1 MW for one hour) are traded as well as ten different standardized *block contracts*. All products are traded simultaneously in two-sided auctions with a *closed* *order book* (sealed bids). Sellers and buyers submit bids on a particular single hour contract that consist of 2 to 64 price/volume pairs within a price scale determined by EEX. Block bids consist of a desired average price and volume per hour for the corresponding block. All bids have to be assigned to one of the *bid areas*, into which the market is divided by EEX.[2]

In the *first price calculation* of the auction for hourly contracts, bids for block contracts are integrated by changing the blocks into price-independent bids for the hours concerned. Then price/volume combinations for every hour are transformed into a sale and a purchase curve of every participant by using linear interpolation. The resulting supply and demand curves are aggregated to a supply and a demand curve for Germany. The price level at the intersection of the two curves is referred to as the *market clearing price*. If no intersection of the demand and supply curves can be determined, EEX may announce a second auction and allow the participants to transmit new bids.

After completion of the *first price calculation*, the average price conditions for block bids are checked. In case not all block bids can be fulfilled, the bid with the highest difference to the average price of the hours concerning the bid is not considered in the *second price calculation*. If the remaining block bids cannot be fulfilled with the market clearing price of the second price calculation, again one bid will be eliminated and a *third price calculation* will take place – and so on until all the remaining block bids can be fulfilled.

In a next step the individual supply and demand curves are aggregated per bid area, resulting in a market clearing price for every bid area. Different prices in the bid areas are adjusted by using price-independent demands and supplies to create power flows from bid areas with low market clearing prices to bid areas with high market clearing prices.[3]

[2] Normally, the bid areas correspond with one or more transmission system operator (TSO) areas, as defined in the Verbändevereinbarung II plus of 13 Dec 2001 (http://www.bmwi.de/Homepage/download/energie/VVStrom.pdf).

[3] The net contractual flow between the TSO areas is calculated by aggregating the individual supply and demand curves per bid area, yielding a market clearing price for each bid area (area price). Price-independent *demands* are introduced to bid areas where the area price is *lower* than the market clearing price, and price-independent *offers* are introduced to bid areas where the area price is *higher* than the market clearing price. This creates a (contractual) power flow between bid areas where the offers and demands introduced sum up to zero. Price levelling out is started in those areas with the highest and lowest area price deviations, respectively. As long as the calculated (contractual) power flow is below the transmission capacity allocated by the TSO to EEX, then the area prices are levelled out completely and the market clearing price becomes effective in all areas. If the transmission capacity is exceeded, in contrast, then a price mechanism is used to relieve the transmission constraints, resulting in bid areas with different bid area prices. Graphically, the supply and demand curves will have to be shifted along the volume axis at a distance equal to the allocated capacity.

2.3.2 Continuous trading

Continuously traded products at EEX (weekdays only) are *base-load* (1 MW delivery over the period midnight – midnight), *peak-load* (1 MW delivery in the period from 8:00 a.m. to 8:00 p.m. on working days), and *weekend base-load contracts* (1 MW delivery for all 48 hours of the weekend). Sellers and buyers submit bids either with or without price limits (*limited orders vs. market orders*) and can add execution conditions and trading restrictions.

At EEX *continuous trading* is preceded by an opening auction and followed by a closing auction. Both auctions are similar to the auction of hourly power except for an *order book balancing phase* in the event of any surplus, i.e. unexecuted orders are made available to the market at the auction price for a limited period of time. At the end of the opening auction, all remaining orders are forwarded to the next possible trading form in accordance with their respective trading restrictions.

In continuous trading, the order book is open, displaying price limits, the accumulated order quantities, and the number of orders for each limit. Each incoming order is immediately checked and executed, if possible, according to price/time priority. The price is the most favourable of all the price limits in the order book, the price limit of the incoming order, and the reference price (which is defined as the last price determined for the product in question).

All orders remaining in the order book at the end of continuous trading participate automatically in the closing auction. Thereafter any outstanding orders are deleted.

2.4 Other Power Exchanges in Europe

Other exchange-based markets for spot trading of electricity in Europe currently comprise (in alphabetical order):

2.4.1 Amsterdam Power Exchange (APX) – The Netherlands

- Day-ahead market
    - operational since May 1999
    - auctions for hourly contracts and complete flexible block contracts
    - 36 participants (as of Jan 2002)
- Adjustment market
    - operational since February 2001
    - continuous trading
    - 17 participants (as of Jan 2002)

2.4.2 Automated Power Exchange UK (APX UK) and UK Power Exchange (UKPX) – United Kingdom

APX UK:

- Operational since March 2001
- Hour-ahead market
  - continuous trading until 4 hours prior to delivery
  - half-hour contracts
  - block contracts
  - 30 participants (as of Nov 2001)

UKPX:

- Operational since March 2001
- Hour-ahead market
  - continuous trading until 4 hours prior to delivery
  - half-hour contracts
  - block contracts
  - 44 participants (as of April 2002)

2.4.3 Borzen – Slovenia

- Operational since January 2002
- Day-ahead market
  - hourly contracts
  - block contracts (base-load, peak-load, off-peak-load)
  - 16 participants (as of April 2002)
- (Week-ahead) 'Preferential dispatch' market
  - base-load
  - peak-load
- Static and dynamic price volatility limits

2.4.4 EXAA – Austria

- Operational since March 2002
- Day-ahead market with hourly contracts
- 13 participants (as of March 2002)
- Market splitting in case of transmission constraints

2.4.5 GME – Italy

- Not yet operational (launch planned for Oct 2002)
- Planned markets
  - day-ahead market
  - adjustment market with two sessions
  - congestion management market
  - reserve market
  - balancing market
- Market splitting in case of transmission constraints

2.4.6 Nord Pool – Scandinavia

- Day-ahead market
  - operational since 1993
  - hours and 5 standardized blocks
- Adjustment market
  - operational since March 1999
  - continuous trading up to 2 hours prior to delivery
- 216 participants (as of Dec 2001)
- Market splitting in case of transmission constraints

2.4.7 OMEL – Spain

- Operational since January 1998
- Day-ahead market
  - simple and complex bids for hourly contracts
- Hour-ahead (adjustment) market
  - simple and complex bids for hourly contracts
  - 6 sessions 135 minutes prior to delivery
- 79 participants (as of Sep 2001)

2.4.8 Powernext – France

- Operational since November 2001
  - Day-ahead market
  - hourly contracts only (so far)
  - 18 participants (as of April 2002)

3 Uncertainties related to bidding

In Deliverable D3.1 “Methodology to identify the relevant uncertainties” uncertainties of interest for CHP operation were discussed. Uncertainty factors of high relevance for the unit commitment and the load dispatch were found to be the electricity price and the heat demand. The influence of the uncertainties of the different parameters can be included in the optimisation models with the help of stochastic optimisation, which has also been described in Deliverable D3.1.

The uncertainty of the heat demand can be taken care of with weather derivatives, that is financial products to hedge against the weather risk. This is probably more relevant for the long-term planning of bids than for the short-term planning.

4 Tools for trading electricity

4.1 Requirements

When a company wants to participate at the spot market either by selling electricity or to make revenue by buying electricity it has to determine at which prices it will sell or buy electricity. The determined prices will certainly depend on the operators unit structure. The price the operator would like to receive for the sold electricity when he/she wants to make a profit depends both on the variable production costs (e.g. fuel costs) and on the start-up costs of the different turbines. It is profitable for him/her to start up a unit if the price at the spot market will be higher than its variable production costs. Mostly the start up of a unit is connected with high start-up costs. Therefore the operator will only start-up its unit, when the price at the spot market is higher than the fuel costs for several hours, so that the revenue in those hours compensates the costs for start-up. The same is true for buying electricity which will result in a shut down of certain units.

To trade electricity at the one day ahead spot market the operator has to submit its bid-curve to the market. describing what amounts of electricity the operator definitely buys or sells at what prices, ranging over a time period of 24 hours divided in time steps of one hour. Therefore the costs for producing a certain amount of electricity has to be known. The production costs have to be determined taking into account all currently available information about the units and the technical requirements. The tool to be developed finally has to determine the bid-curve by relating the prices for selling or buying certain amounts of electricity for each hour of one day.

From a technical point of view, the tool shall export the bid-curve into a table sheet (e.g. a sheet in Microsoft Excel format), enabling the operator to view this function in graphical form. For certain prices the amount of electricity sold or bought can be seen. This is illustrated in Figure 4.1. The column to the right shows the possible spot prices and the column for each hour then shows the amount of electricity that can be sold or bought, a negative value indicating buying, in that hour for the corresponding price. The bid function has to be submitted 12 hours before the next day. Since the operator wants to include the newest information about its units the requirement for the calculation time should be within one hour.

22.04.2002	**offer curve for spot market**										
Prices in EUR/MWh		**6.6**	**9.0**	**15.4**	**21.5**	**17.0**	**23.5**	**27.0**	**34.0**	**35.0**	**remarks**
Hourly bids	**00:00-06:00**	-350	-300	-200	100	200	350	400	430	530	
	06:00-10:00	-400	-400	-300	0	100	300	400	430	530	
	10:00-12:00	-500	-450	-350	-50	0	200	300	300	400	
	12:00-14:00	-550	-450	-400	-150	-30	180	280	280	380	
	14:00-17:00	-500	-450	-360	-150	-50	100	200	370	420	
	17:00-19:00	-500	-450	-360	-200	50	100	250	370	420	
	19:00-21:00	-450	-400	-320	-100	50	150	280	370	430	
	21:00-22:00	-430	-400	-250	-100	100	200	350	370	530	
	22:00-00:00	-400	-350	-200	100	180	350	400	430	530	

Figure 4.1: Table sheet for the bid-curve

4.2 Deterministic model

The optimisation model developed in WP1, 2 and 3 for the long term optimisation includes the possibilities to buy and sell electricity at the spot market. It is therefore also a tool for trading electricity. The deterministic version of the model use the spot market price for every hour as an input. With a function for the expected electricity price on the spot market the model can then be used for the planning of when to buy and when to sell electricity to the spot market. By vary the spot price for a specific period of time, different amounts of sold and bought electricity at that time can be calculated.

Several types of contracts will be included for the selling and buying of electric power. In order to reduce complexity all sales and purchases will be aggregated to certain standard contracts. As role models we will use the standard products of the LPX market (e.g. peak and base contracts for weeks, months, quarters, years).

4.3 Stochastic unit commitment model

Written by Heike Brand, IER

When it comes to short-term bidding, for example placing bids at the one day ahead market, the uncertainty factors can become very important. Then is it recommended to use a stochastic optimisation model, a model including the uncertainties, to plan the bids especially when turbines with high start-up costs are considered. Because of the high volatility of the prices it is not obvious when to start up or shut down a special unit if high costs are involved.

The procedure based on the deterministic model described above does not take into account these price variations. Only statements about possible changes of the unit commitment schedule for singleton turbines can be made. Considering all possible combination of improvements for the unit commitment is not possible.

In the following, a procedure is described taking into account the uncertain and varying LPX-prices. The method of stochastic optimisation is used in order to determine a robust bid curve resulting in the highest revenues *in average*, and not for special situations. The advantage of this solution found by using stochastic optimisation compared to other heuristic procedures presented in /Hobbs et al. 2000/ is that all possible solutions are considered weighted according to their possibility of realisation. Of course, for reliability, a good estimation of the LPX-prices is the crucial point that however, will not be addressed here.

This new procedure will determine a bid function for every hour depending on the price level of this hour. While determining this function the procedure takes into account the possible realisations of the LPX-prices during the other hours. The crucial point of this procedure is to adjust the optimisation model by demanding new restrictions: the amount of electricity to be sold or bought at LPX should be the same if - independent of the actual scenario - the LPX price lies within the same of many a priori fixed price levels.

The procedure consists of the following steps being described in detail below.

1) Scenario generation using Monte Carlo simulations
2) Creation of the scenario tree based on these scenarios and its reduction
3) Setting up the stochastic optimisation model

4) Determination of the bidding curve out of the solution of the stochastic optimisation problem

1) Scenario generation by Monte Carlo simulations

In order to use the information of the time series of the LPX-market prices, historical data is analysed and the parameters of the price process are determined assuming a known probability distribution for the stochastic parameter modelling the LPX price. Monte Carlo simulation is used as not enough data is available to create scenarios out of historical time series. Further scenario reduction algorithms are applied in order to find a subset of scenarios which is a good representative set of possible scenarios yielding a much smaller scenario tree greatly reducing the effort for the computational determination of the solution.

2) Creation of the scenario tree and its reduction

To use these scenarios for stochastic optimisation we have to construct a tree, the root of the tree being the price known for the last hour of the day before. How this is done in detail, is described in /Dupacova et al. 2000/. Before the tree is created, different price levels have to be defined as intervals of price values. The number of price levels is an input parameter indicating on the one hand the precision of the modelling of the LPX prices by this stochastic optimisation model, while clearly, on the other hand determining its size and thus the computational effort for finding its solutions. The intervals can be taken as equidistant division of the whole interval of prices. The boundaries of these price intervals will later be used for the creation of the bid function. At those bounds the bid function can jump from one quantity of offered electricity to the next. The number of price levels are often chosen to be the number of electricity producing units since the turn on or off of one unit corresponds to a jump in the maximum amount of electricity that can be produced. A negative value of the offer indicates that electricity is bought while a positive value indicates that it is sold. In Figure 4.2, 7 intervals have been chosen, with the boundaries 10 €/MWh, 20 €/MWh, ..., 70 €/MWh. When the price is in the range of [30 €/MWh, 40 €/MWh], then 100 MWh are offered. In this example the quantity of bought electricity is the same for the two price levels/intervals [0 €/MWh, 10 €/MWh] and [10 €/MWh, 20 €/MWh]. Also for the price levels/intervals [40 €/MWh, 50 €/MWh] and [50 €/MWh, 60 €/MWh] the same amount of electricity is offered to be sold.

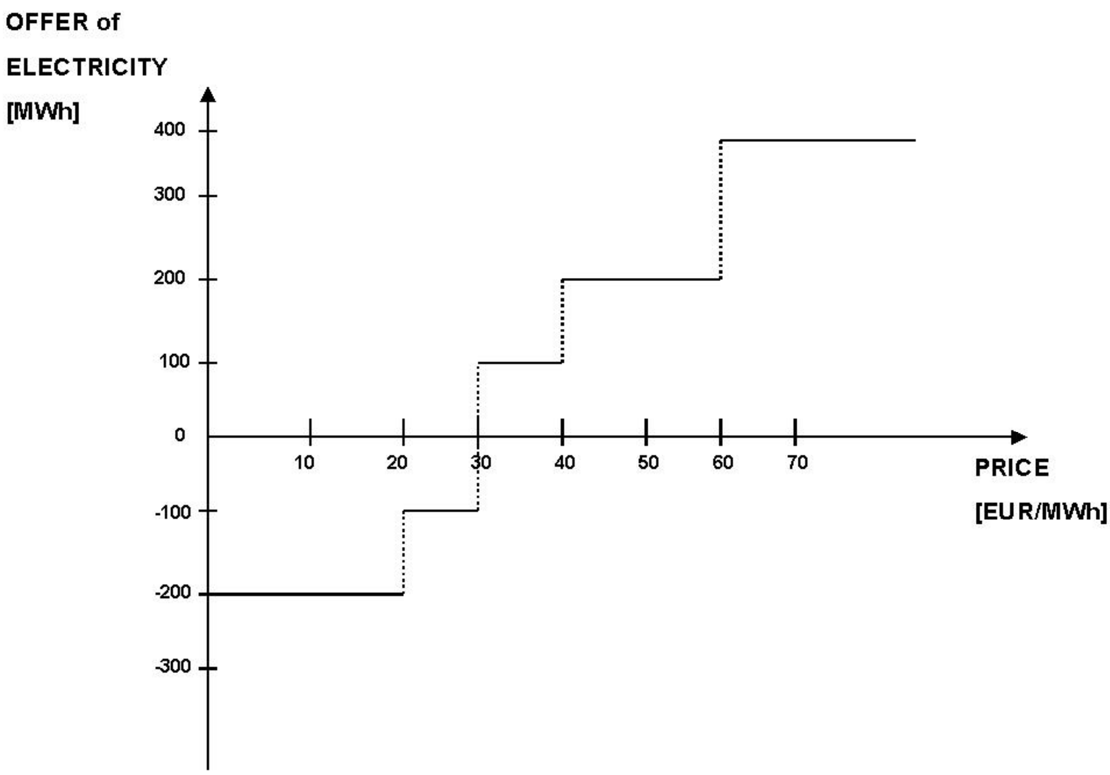

Figure 4.2: An example of price levels/intervals for different offers of electricity

The number of price levels determines the upper bound for the number of successors of each inner node of the tree to be constructed. It is required that at each stage the number of nodes at this stage has to be greater or equal to the number of price intervals.

$$number\_of\_nodes(stage\_i) \geq number\_of\_price\_levels$$

As the number of nodes increases with increasing stage number, the number of branches at each stage can be reduced and still obtaining a detailed modelling of the varying prices. One has to be aware of the fact, that the probabilities for the different branches depend on the actual and past realisations of the prices according to the assumed probability distribution. For each time step of one hour one would ideally choose a new stage. This would result in a tree with at least 2^{24} leaves indicating that for computational reasons it will be necessary to group certain hours. The proposal is to merge hours accordingly to the LPX-price statistics which results in 6 stages for the first day and an additional stage for the following day. An example of a tree is given in Figure 4.3. In the first stage the tree branches into 8 successors because 8 price levels have been chosen, from stage 2 until stage 6 the tree branches into 3 successors and in the last stage only 2 successors are defined, so that a tree with 8*3*3*3*3*3*2 = 3’888 leaves is defined.

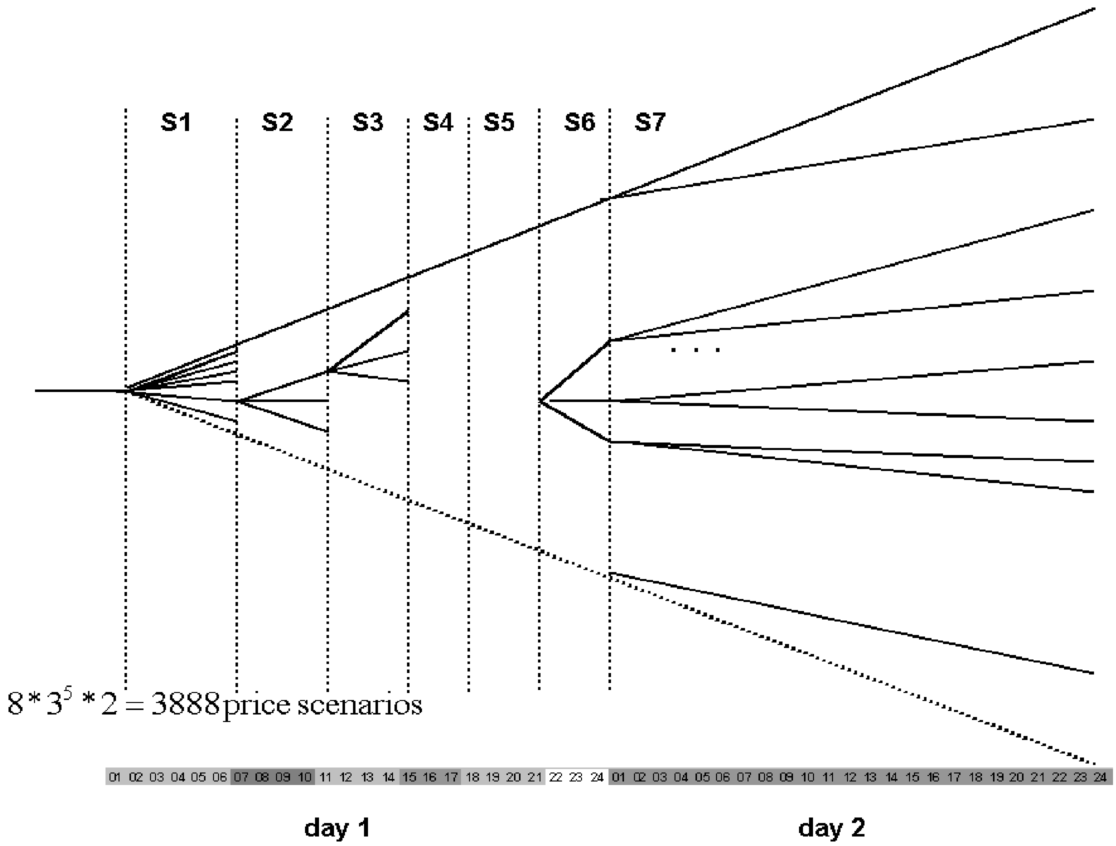

Figure 4.3: An example of a scenario tree

3) Setting up the stochastic optimisation model

The bidding curve to be determined can not depend on the actual scenario that will be realised. On the other hand, if the price level in different scenarios is the same, then these scenarios refer to the same point on the bidding curve, relating a certain amount of electricity to be sold or bought to this price or price level. This means, that it is the price level that the bidding curve is depending on. As it is difficult to estimate the LPX prices very well, we have to determine one bid curve, independently of which scenario will occur. This can be done with the stochastic programming model developed in WP3. As for each hour the amount of electricity the operator wants to sell or buy has to be determined, and as the operator has to take the decision before he knows the spot market prices and cannot postpone the decision, the restriction that for a certain price, P, the sold or bought amount has to be equal for all scenarios, where this price, P, will be realised is added to the model. This restriction guarantees that independent of the scenarios the amount sold or bought at a certain price is calculated, and that this amount is independent of the scenario that will occur. But note: when determining this amount all possible price scenarios for the hours before and after that certain hour are taken into consideration.

Thus the important step is to add the constraint (1) about the amount of electricity to be sold or bought at the LPX market to the optimisation model. In advance, different price intervals have to be defined, in the following referred to as price levels. If, the prices for two different scenarios lie within the same price level (i.e. within the same of the previously defined intervals), then the additional restriction is given by the equality of the amounts of electricity sold or bought in these two scenarios.

$$P_{\mathrm{EL}}^{\mathrm{BID}}(s_1,t) = P_{\mathrm{EL}}^{\mathrm{BID}}(s_2,t) \qquad \forall s_1, s_2 \in SCENARIOS_{interval(t)} \quad (1)$$

with $SCENARIOS_{interval(t)} = \left\{ s \in SCENARIOS \mid price_s^{\mathrm{SPOT}}(t) \in \left[b_{interval(t)}^{\mathrm{LEFT}}, b_{interval(t)}^{\mathrm{RIGHT}} \right] \right\}$

and

$P_{\mathrm{EL}}^{\mathrm{BID}}(s,t)$: electricity offered in scenario s at time t

$price_s^{\mathrm{SPOT}}(t)$: price of electricity at spot market in scenario s at time t

$b_{interval(t)}^{\mathrm{LEFT}}$: left bound of the price intervall of interest at time t

$b_{interval(t)}^{\mathrm{RIGHT}}$: right bound of the price intervall of interest at time t

The amount of electricity sold at the different price levels for each time step will be denoted as $P_{\mathrm{EL},interval(t)}^{\mathrm{BID}}$.Taking into account these additional constraints guarantees that indeed one price level is set in relation to the amount of electricity to be traded at the LPX market in the bidding curve.

4) Determination of the bidding curve out of the solution of the stochastic optimisation problem

Having found the solution of the optimisation problem with the additional constraints, the bidding curve can be determined. The solution contains the amount of electricity sold and bought for each node. And for each node, the LPX price is also known since this is the base for the branching of the tree. Thus the bidding curve can quite easily be constructed by relating the amount of electricity sold or bought at each node of each stage of the scenario tree to the price of these nodes. The additional restrictions in the optimisation model by construction just forced the amounts for these nodes to be equal, if the prices all belonged to the same price level.

The last task is to determine those prices at which the bid function jumps to a higher amount of offered electricity (amount of electricity sold). One possibility is to chose the left bounds $b_{interval(t)}^{\mathrm{LEFT}}$ of the intervals as the points where the bid function can jump to a new electricity offer (correspondingly $b_{interval(t)}^{\mathrm{RIGHT}}$ can be chosen for the demand curve (the amount of electricity bought)). This method will underestimate the costs of electricity production, because it will offer the amount of electricity $P_{\mathrm{EL},interval(t)}^{\mathrm{BID}}$ at the lowest price of each interval. As the scenario tree does not represent all possible scenarios, it cannot be guaranteed that this lower bound is included in the scenario tree and so no statement can be made about the amount of electricity sold at that lower bound. For that reasons it is better to chose the lowest of the scenario prices belonging to that price level *intervall(t)* as the new boundary of the intervall.

$$\hat{b}^{LEFT}_{interval(t)} = min\{price^{SPOT}_{s}(t) \mid s \in SCENARIOS_{interval(t)}\}$$

with

$$SCENARIOS_{interval(t)} = \{s \in SCENARIOS \mid price^{\mathrm{SPOT}}_{s}(t) \in [b^{\mathrm{LEFT}}_{interval(t)}, b^{\mathrm{RIGHT}}_{interval(t)}]\}$$

$\hat{b}^{LEFT}_{interval(t)}$:left boundary for price intervall *interval(t)* for offer curve

$price^{SPOT}_{s}(t)$: price of electricity at spot market at time t in scenario s

This alternative of creating the bid function reduces the risk that electricity is sold below its production costs. Regarding the determination of the demand curve the operator only wants to buy if the price is below a certain level. Therefore we have to adjust the right bounds of the price intervals. Similar to the above procedure for the offer curve, we chose as right bound the maximum of the prices occurred within that price level.

$$\hat{b}^{RIGHT}_{interval(time)} = max\{price^{SPOT}_{s}(t) \mid s \in Scenario_{interval(t)}\}$$

with :

$$SCENARIOS_{interval(t)} = \{s \in SCENARIOS \mid price^{\mathrm{SPOT}}_{s}(t) \in [b^{\mathrm{LEFT}}_{interval(t)}, b^{\mathrm{RIGHT}}_{interval(t)}]\}$$

$\hat{b}^{RIGHT}_{interval(t)}$:right boundary for price intervall *interval(t)* for demand curve

$price^{SPOT}_{s}(t)$: price of electricity at spot market at time t in scenario s

In Table 4.1, Table 4.2 and Figure 4.4 the results for an example with a tree with 8 branches at the first stage and 3 branches at the second stage and 2 branches at the following 6 stages are given. The number of price levels are set to 8. For eight stages (as described in Figure 4.3) we then get $8*3*2^6 = 1536$ scenarios. At the 2nd stage, representing the hours 11-14, we have 24 different scenarios. The prices and amount of electricity sold or bought for each scenario resulting from the stochastic optimisation model is shown in Table 4.1. A negative value of the amount indicates that it is bought. It can be seen that for each node where the spot price belongs to the same price interval the same amount of electricity is sold or bought.

Table 4.1: Prices for the different scenarios at the stage representing the hours 11-14 for the example described in the text

Spot price at the stage for hour 11:00-14:00	Belongs to price interval	Amount of electricity (MWh)
15.1	14.3-16.58	-50
27.54	25.70-27.98	570
18.77	16.58-18.86	100
16.28	14.3-16.58	-50
14.58	14.3-16.58	-10
21.26	21.14-23.42	370
25.98	25.70-27.98	570
30.26	27.98-30.26	650
25.51	23.42-25.70	470
26.29	25.70-27.98	570
13.24	12.02-14.3	-120
26.09	25.70-27.98	570
18.98	18.86-21.14	250
13.22	12.02-14.3	-120
20.91	18.86-21.14	250
16.13	14.3-16.58	-50
22.36	21.14-23.42	370
14.53	14.3-16.58	-50
20.15	18.86-21.14	250
23.94	23.42-25.70	470
19.13	18.86-21.14	250
16.56	14.3-16.58	-50
12.02	12.02-14.3	-120
17.18	16.58-18.86	100

By choosing the highest price in the interval for buying electricity and the lowest price in the interval for selling we get the bid function shown in Table 4.2, which is illustrated in Figure 4.4. In the figure also the price intervals given in the optimisation are shown. It can be seen that by choosing the lowest price in the interval for selling electricity the jump to a higher amount sold is made at a higher price than in the original price interval and then reducing the risk that electricity is sold below its production costs as described above. By choosing the highest price in the interval for buying electricity the jump to a lower amount bought is made at a lower price than in the original price interval then reducing the risk that electricity is bought at prices higher than the production costs.

Table 4.2: Bid function for the hours 11-14 for the example described in the text

Price	0-13.24	13.24-15.1	15.1-17.18	17.18-18.98	18.98-21.26	21.26-23.94	23.94-25.98	25.98-30.26	>30.26
Sold amount	-120	-10	0	100	250	370	450	570	650

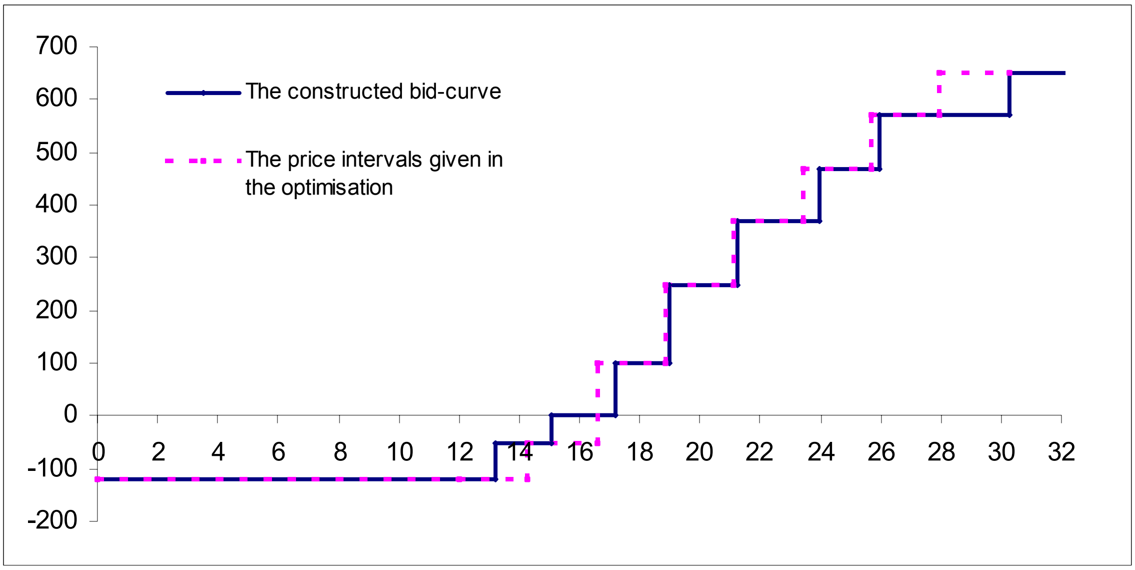

Figure 4.4: Illustration of the bid curve for the example described in the text

4.4 Real Option Model

4.4.1 Description

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One area of application for the real option model partly developed in WP3 is to provide a financial model for generation assets. The idea is to view generation assets as one part of the portfolio of a utility and using the model of the generation assets is then one part of measuring and managing the risk situation of the whole utility portfolio and also to optimise this portfolio. The real option model developed in WP 3 is based on a mixture of backwards stochastic dynamic programming and forward Monte Carlo simulation. Assuming that the electricity spot market is liquid it is not necessary to solve the commitment problem jointly for all production assets. Instead it is always optimal to run production capacity for each turbine against the spot market. The solution of such a problem can be represented by so-called *indifference loci* for all periods over the optimisation horizon. Figure 4.5 shows an example solution. The line marked with triangles is the turn-on barrier, that is, if the turbine is off and the observed spot price exceeds this line, then it is optimal to turn it on. The line with squares is the turn-off barrier, that is, if the turbine is on and the spot price falls below this line, then it is optimal to turn it off. One should note that, because of start-up times and minimum up times, it is not the observed spot price which determines the profit of the unit, but the *conditional expectation of the spot price* over the following periods. Therefore, the indifference loci move according to the changes in the conditional expectation of spot prices. For example, at night the expected spot price is so low that it requires a very high price to justify turning on. At the same time the turn-off barrier is also very high, meaning that if the turbine is on, it does not take much reduction in the spot price to make it optimal to turn it off. The single most important input to the model is a stochastic model for the evolution of electricity spot prices.

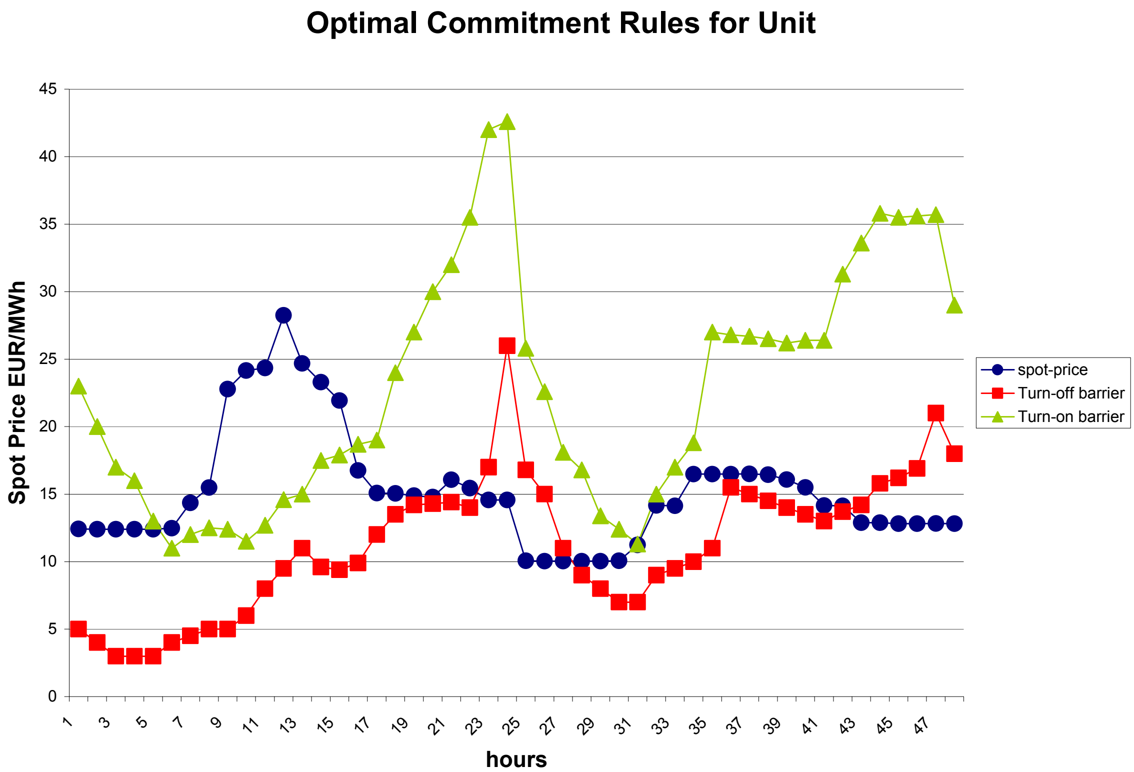

Figure 4.5: An example solution from the real option model

The real option model can be extended to other products that the unit could sell, e.g. ancillary services or emission rights. This allows owners of generation assets to exercise their multi-commodity options across various markets in a consistent and profit maximizing way. For CHP plants an extension for heat production is necessary.

4.4.2 The use of the real option model as a bidding tool

The results from the real option model could also be used for constructing a bidding curve.

The predicted spot price and the indifference loci gives the unit commitment for each hour. The predicted on-off status for a turbine the hour before the hour for which the bid curve are to be constructed then gives the border for the bidding price. If it is on the turn-off line gives the border for the bid price for that turbine, if it is off instead the turn-on line gives the border. That is, if the turbine is on at time *t-1* the turbine will produce the maximum amount of electricity at time *t* if the spot market price is above the turn-off line at time *t*. Otherwise the production will be zero. If the turbine is off at time *t-1* the maximum amount of electricity will be produced only if the spot market price is above the turn-on line. The price level above which the unit will produce electric power can then be expressed as below.

$$price_u^{SPOT}(t) = price_u^{TURN-OFF}(t)O(t-1) + price_u^{TURN-ON}(1-O(t-1))$$

with:

$price_u^{SPOT}(t)$:the spot market price level for unit u at time *t*

$price_u^{TURN-OFF}(t)$:the price for the turn-off line for unit u at time *t*

$price_u^{TURN-ON}(t)$:the price for the turn-on line for the unit u at time t

$O_u(t-1)$:the on-off status for unit *u* at time *t-1*

To construct a bidding curve we now have to relate a certain amount of offered electricity to a certain price interval for each hour. The bounds of the intervals is chosen with the help of the determined price levels for the different turbines, with the right bound for the first interval equal to the price level for the turbine with the lowest price. That is, we get a jump in the offered electricity for each new turbine that is turned on (or not turned off). The amount of sold or bought electricity for each price interval is calculated by summing up the electricity produced by the units in that price interval and withdraw the electricity demand.

$$P_{\mathrm{EL},interval(t)}^{\mathrm{BID}} = \sum_{u_{ON}} P_{MAX,EL,u} - P_{EL,DEMAND}(t) \qquad \forall u_{ON} \in UNITS_{interval(t)}$$

with $UNITS_{interval(t)} = \{u \in UNITS \mid price_u^{\mathrm{SPOT}}(t) \in [b_{interval(t)}^{\mathrm{LEFT}}, b_{interval(t)}^{\mathrm{RIGHT}}]\}$

and

$P_{\mathrm{EL},interval(t)}^{\mathrm{BID}}$:the amount of electricity offered for the interval of interest at time *t*

$P_{MAX,EL,u}$:maximum electric power that can be produced by unit *u*

$P_{EL,DEMAND}(t)$: electricity demand at time *t*

$b_{intervall(t)}^{\mathrm{LEFT}}$: left bound of the price interval of interest at time *t*

$b_{intervall(t)}^{\mathrm{RIGHT}}$: right bound of the price interval of interest at time *t*

5 Literature

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